RESEARCH ARTICLE



Restricted dynamic consistency

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Abstract

Dynamic consistency is a key behavioral property in dynamic economic models, making it possible to use tractable dynamic programming techniques. However, when combined with the separation of time and risk preferences, it can lead to unrealistic predictions that contradict empirical evidence. This paper demonstrates that dynamic consistency can be relaxed to hold over a much smaller domain of consumption programs while maintaining sufficient richness for practical applications and allowing the separation of risk aversion from intertemporal substitution. As an application, I introduce a new model of dynamic preferences, the Epstein–Zin–Selden–Stux preferences. These preferences are recursive only within a restricted domain. Recent experimental results by Meissner and Pfeiffer (J Econ Theory 200:105379, 2022), which Epstein– Zin preferences cannot rationalize, find a natural explanation through this new model. Finally, I consider an application of this new model to a consumption-investment problem.

Keywords Recursive utility \cdot Dynamic consistency \cdot Intertemporal substitution \cdot Risk aversion

JEL Classification C61 · D81

1 Introduction

Dynamic consistency is a key property of dynamic economic models. Along with other standard axioms, it implies that intertemporal utilities satisfy a set of recursive conditions, which permits solving models by means of standard dynamic programming techniques. Models of recursive utility allow for the disentangling (e.g., see Chew and Epstein 1991) of risk aversion from the elasticity of intertemporal substitution (EIS), a

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common desideratum in many economic applications. Moreover, dynamic consistency is an appealing axiom from a normative perspective [e.g., see Ghirardato (2002) or Al-Najjar and Weinstein (2009)]. Because of these facts, even if dynamic consistency is often violated in experiments, it is an assumption of central importance in many economic settings, from asset pricing (Epstein and Zin 1991; Suzuki and Yamagami 2024) to optimal fiscal policy (Karantounias 2018).

However, for recursive preferences, disentangling risk aversion from intertemporal substitution is incompatible with indifference to the timing of resolution of uncertainty (Kreps and Porteus 1978). This fact poses a problem from a normative point of view, as it implies that non-instrumental information can be valuable. Moreover, a preference for early or late resolution of uncertainty can create empirical issues.

For instance, Epstein et al. (2014) argue that standard parameter assumptions in finance and macroeconomic applications of the Epstein–Zin model-the cornerstone of recursive utility-lead to implausibly large premia for early resolution of consumption risk. They introduce the concept of a *timing premium*, defined as the proportion of wealth a decision-maker is willing to forgo to resolve all consumption uncertainty immediately. Their findings show that common parameter values used in the literature result in timing premia exceeding 30%, a figure difficult to justify at the level of introspection.

This result contrasts with the finding of Meissner and Pfeiffer (2022) who in their experiment find that on average subjects have moderate preferences for early resolution with average timing premium about 5%, and around 40% of subjects in the experiment are indifferent to the timing of resolution of uncertainty. Moreover, they find that under estimated Epstein–Zin preference parameters, the predicted timing premium is small and negative. This discrepancy arises because their estimates of preference parameters indicate that relative risk aversion is strictly smaller than the reciprocal of the EIS—a condition under which the Epstein–Zin model predicts a positive timing premium.

In this paper, I demonstrate that the axiom of dynamic consistency can be relaxed such that it does not apply to all consumption programs. This weakening still allows preferences to disentangle risk aversion from intertemporal substitution. Simultaneously, it helps to reconcile experimental findings: under certain specifications, this approach can imply a moderate positive or zero timing premium, even when relative risk aversion is less than the reciprocal of the EIS.

Preview of results

I consider preferences defined over the set of all conceivable consumption programs. These preferences satisfy the axiom of dynamic consistency over only a strict subset of consumption programs, whose only requirement is to be topologically connected. Notably, I provide examples of consumption domains that are not only topologically connected but also *relevant* in applications.

First, I show that despite this *restricted* notion of the dynamic consistency axiom, a recursive representation can still be obtained that applies over the restricted consumption domain (Theorem 1). Moreover, I show that consumption domains of this kind, are nevertheless, rich enough to allow for disentangling risk aversion from intertemporal

substitution (Proposition 2). As a consequence, one can separate risk aversion from intertemporal substitution under this weaker notion of dynamic consistency, while at the same time considering a consumption domain that is relevant in applications.

Relevant domains of consumption. I consider two main examples of relevant domains of consumption (see Sect. 3). The first is the domain GR of consumption programs that resolve gradually. To illustrate, in consumption-savings applications, consumption at every period is a function of income whose uncertainty resolves gradually. The second is the domain IND of programs that are independently distributed over time. For instance, in consumption-based asset pricing models, one of the main variables of interest is log consumption growth, which is often modelled as an i.i.d. process. The main technical contribution is to show that both these domains are connected, as required by Theorem 1 (see Proposition 1).

Epstein–Zin–Selden–Stux preferences. These general theoretical results are then specialized to the Epstein–Zin case. Building on Theorem 1, Theorem 2 provides an axiomatic foundation of a novel model that "merges" the Epstein–Zin preferences (Epstein and Zin 1989) with the dynamic ordinal certainty equivalent (DOCE) model of Selden (1978), an alternative approach to Epstein–Zin preferences intended to separate risk aversion from intertemporal substitution (see also Selden and Stux 1978 for the axiomatization with finitely many arbitrary time periods).

Unlike Epstein–Zin preferences, DOCE preferences are characterized by indifference to the timing of resolution of uncertainty [see the discussion in Kubler et al. (2019)]. I refer to these novel "hybrid" preferences as Epstein–Zin–Selden–Stux (EZSS). EZSS preferences are represented by a utility function which is provided by the standard Epstein–Zin formulation for consumption programs on the restricted domain of consumption, while for consumption programs not in this domain it takes the DOCE formulation.

Application: timing premia. Meissner and Pfeiffer (2022) estimate the preference parameters of the Epstein–Zin (EZ) model and find that

relative risk aversion
$$< \text{EIS}^{-1}$$
. (1)

A similar pattern is observed in the dynamic discrete choice model estimated by Lu et al. (2024). Recall that Epstein–Zin preferences predict that when equation (1) holds the timing premium should be negative—which contradicts the experimental evidence of a positive but moderate timing premium. By contrast, under DOCE preferences, the timing premium would always equal zero.

I show that EZSS preferences can reconcile this empirical evidence (see Sect. 5.1). Specifically, when the relevant domain is IND, the parameters estimated in their experiment—which satisfy (1)—yield a timing premium of approximately 4%. This reflects a moderate preference for early resolution of uncertainty, in line with the elicited value of the premium. Observe that neither DOCE nor Epstein–Zin preferences can explain this evidence. On the other hand, if the relevant domain is GR, the timing premium equals zero, consistent with the behavior of 40% of the subjects in the experiment.

As a direct consequence of this result, I show that in a stylized portfolio problem, EZSS preferences lead to investment in risky assets during the initial period and discourage further risky investment later on-even if condition (1) holds. Consequently, the optimal consumption plan resolves early, at the initial stage. This consumption pattern cannot arise under either the Epstein–Zin or DOCE models, which predict late and gradual resolution of consumption, respectively, under the same condition. This finding highlights the distinct empirical predictions of the EZSS model.

Related literature

The theoretical paper closest to the present one is Kubler et al. (2019). One of their major contributions is to construct a restricted set of consumption programs such that DOCE preferences satisfy time consistency. Because these preferences always satisfy the separation of time and risk preferences and indifference to the timing of risk resolution, they are able to obtain preferences that on this restricted domain satisfy all three of these important properties combined. I adopt a similar approach, which, however, differs from theirs in several ways. From a technical perspective, the assumption on the domain they consider (ICER) is based on a condition about the distribution of asset returns, while I consider a general topological assumption. More substantially, the preferences I axiomatize in Theorem 2 "mix" DOCE preferences and Epstein–Zin in such a way that they are compatible with a moderate preference for early resolution of uncertainty even if relative risk aversion is greater than the reciprocal of EIS, a feature which neither DOCE nor Epstein–Zin preferences can have.

The concept of timing premium has also been applied to retirement and social security (Caliendo et al. 2023). Andreasen and Jørgensen (2020), building on the results of Epstein et al. (2014), propose a generalization of Epstein–Zin preferences to address existing puzzles in asset pricing. However, their model still predicts an excessively high timing premium.

My work also draws from Johnsen and Donaldson (1985), who provide an axiomatic representation of recursive preferences in a setting with two periods. In particular, Theorem 1 extends their result to multiple periods, with the novelty that dynamic consistency can hold only on a strict subdomain of preferences. Hammond (1989) argues that the work of Johnsen and Donaldson contains a hidden assumption which restricts the domain of consumption, therefore explaining the apparent contradiction with the work of Weller (1978) showing that dynamic consistency, along with other standard conditions, implies expected utility behavior. Epstein and Le Breton (1993) also link dynamic consistency to expected utility. In the present paper, the filtration which describes the evolution of information is taken to be fixed, a standard assumption in applications; therefore their results do not apply.

Both Sarin and Wakker (1998) and Epstein and Schneider (2003) under an assumption of monotonicity of preferences characterize recursive multiple prior preferences (see also Wakai 2015; Amarante and Siniscalchi 2019). Bommier et al. (2017) and Marinacci et al. (2023) show that this assumption of monotonicity in general has strong restrictions on recursive preferences. I consider a formal setting of *uncertainty* [unlike a setting of *risk* such as in Kreps and Porteus (1978) or Epstein and Zin (1989)]. This setting permits considering recursive models of ambiguity aversion, which are well known to be relevant in applications [e.g see Ju and Miao (2012) or Collard et al. (2018)].

2 Preliminaries

2.1 Framework

Time is discrete and varies over a finite horizon $t \in \{0, 1, ..., T\} \equiv T$. The information structure is described by a filtered space $(\Omega, \{\mathcal{G}_t\}_{t\in T})$ where Ω is an arbitrary set of states of the world and $\mathcal{G} = \{\mathcal{G}_t\}_{t\in T}$ is a sequence of σ -algebras such that $\mathcal{G}_0 = \{\Omega, \emptyset\}$ that satisfy $\mathcal{G}_t \subset \mathcal{G}_{t+1}$ for t = 0, ..., T - 1.¹ For simplicity, assume that every algebra $\mathcal{G}_t, t \in T$, is generated by a finite partition, where $\mathcal{G}_t(\omega)$ denotes the element of the partition containing $\omega \in \Omega$. The assumption of finiteness is not necessary but avoids the need to formally establish the existence of each recursive utility model considered in this paper.²

Let *X* denote the set of outcomes, which is assumed to be a convex subset of \mathbb{R}^n . The main case we are interested in is $X = \mathbb{R}_+$. An act or consumption program is an *X*-valued, *G*-adapted process, that is, a sequence $(f_t)_{t \in \mathcal{T}}$ such that $f_t : \Omega \to X$ is \mathcal{G}_t -measurable for every $t \in \mathcal{T}$. \mathcal{F} is the set of all consumption programs. \mathcal{D} is the set of deterministic consumption programs, $d = (d_0, d_1, \ldots, d_T) \in \mathcal{D}$ if and only if d_t is measurable w.r.t. \mathcal{G}_0 for all t. Since each \mathcal{G}_t is finitely generated, the set of all \mathcal{G}_t -measurable acts can be endowed with the product topology. It follows that we can endow \mathcal{F} with the product topology.³ Under this topology, \mathcal{F} is a metrizable separable space, and therefore any subset of $\mathcal{R} \subseteq \mathcal{F}$ is also a metrizable separable space.

Given a measurable space (S, Σ) and a subset $K \subseteq \mathbb{R}$, let $B_0(\Sigma, K)$ denote the set of simple Σ -measurable functions with range contained in K. A function I : $B_0(\Sigma, K) \to \mathbb{R}$ is (i) continuous if it is continuous in the sup-norm topology; (ii) monotone if $\xi(s) \ge \xi'(s)$ for every $s \in S$ implies $I(\xi) \ge I(\xi')$; (iii) strictly monotone if it is monotone and $\xi(s) \ge \xi'(s)$ for every $s \in S$ with one strict inequality implies $I(\xi) > I(\xi')$; and (iv) normalized if I(x) = x for every $x \in \mathbb{R}$ (where x denotes the constant function $x1_{\Omega}$). Call I a *certainty equivalent* if it is continuous, strictly monotone, and normalized. Given a probability measure P defined on (S, Σ) , an expected utility functional is given by $\mathbb{E}_P \xi = \int \xi(s) dP(s)$.

The primitives of interest are a family of \mathcal{G} -adapted weak orders (complete and transitive relations) $\{\succeq_{t,\omega}\}_{(t,\omega)\in\mathcal{T}\times\Omega}$ on \mathcal{F} .⁴ Let \succeq_0 denote the preference at time

 $\{f: \Omega \to X: f \text{ is measurable w.r.t. } \mathcal{G}_t\},\$

can be identified with a subset of $\mathbb{R}^{|\mathcal{G}_t|}$, and therefore \mathcal{F} can be endowed with the product topology.

⁴ By \mathcal{G} -adapted I mean that $\succeq_{t,\omega} = \succeq_{t,\omega^*}$ whenever $\mathcal{G}_t(\omega) = \mathcal{G}_t(\omega^*)$.

¹ See Stokey and Lucas (1989) for canonical interpretations of this setting in terms of shocks/observations.

 $^{^2}$ See Marinacci and Montrucchio (2010) for a thorough treatment of the topic.

³ Denoting with $|\mathcal{G}_t|$ the number of elements of the partition that generates \mathcal{G}_t , the set

zero. For brevity, I typically denote the collection of preferences $\{\succeq_{t,\omega}\}_{(t,\omega)\in\mathcal{T}\times\Omega}$ with $\succeq_{t,\omega}$ only. Say that a collection of real-valued functions $(V_t(\omega, \cdot))_{t,\omega}$, where for every $t \in \mathcal{T}$, $V_t : \Omega \times \mathcal{R} \to \mathbb{R}$ and $\mathcal{D} \subseteq \mathcal{R} \subseteq \mathcal{F}$, represents $\succeq_{t,\omega}$ over \mathcal{R} if for every $h, h' \in \mathcal{R}$

$$V_t(\omega, h) \ge V_t(\omega, h') \iff h \succeq_{t,\omega} h' \text{ for every } (t, \omega).$$

A collection $(I_{t,\omega})_{t,\omega}$ of certainty equivalents is adapted to $(V_t(\omega, \cdot))_{t,\omega}$ and \mathcal{G} if each $I_{t,\omega}$ is defined on the set $\{\xi \in B_0 (\mathcal{G}_{t+1}, V_{t+1}) : \xi = V_{t+1}(\cdot, f), f \in \mathcal{R}\}$ and satisfies $I_{t,\omega} = I_{t,\omega^*}$ when $\mathcal{G}_t(\omega) = \mathcal{G}_t(\omega^*)$.

Recursive preferences are based on the notion of a *time aggregator*. Given $A \subseteq \mathbb{R}$, a function $W : X \times A \to \mathbb{R}$ is a time aggregator if it is continuous and strictly increasing in the second variable.

2.2 Recursive preferences

I consider a general notion of recursive representation of preferences.

Definition 1 Let \mathcal{R} satisfy $\mathcal{D} \subseteq \mathcal{R} \subseteq \mathcal{F}$. Preferences $\succeq_{t,\omega}$ admit a recursive representation over \mathcal{R} if and only if there exist $(V_t(\omega, \cdot))_{t,\omega}$ that represent $\succeq_{t,\omega}$ over \mathcal{R} such that $V_t(\omega, \mathcal{R}) = V_t(\omega', \mathcal{R}) \equiv V_t \subseteq \mathbb{R}$ for every $\omega, \omega' \in \Omega$, $V_T(\omega, h) = u(h_T(\omega))$ for every $\omega \in \Omega$, and for every $0 \le t < T$,

$$V_t(\omega, h) = W\left(h_t(\omega), I_{t,\omega}\left(V_{t+1}(\cdot, h)\right)\right) \text{ for every } \omega \in \Omega \text{ and } h \in \mathcal{R},$$
(2)

where

- (i) $u: X \to \mathbb{R}$ is a continuous function that satisfies u(z) = 0 for some $z \in X$;
- (ii) $(I_{t,\omega})_{t,\omega}$ is a collection of certainty equivalents adapted to $(V_t(\omega, \cdot))_{t,\omega}$;
- (iii) $W: X \times \bigcup_{\tau \ge t+1} V_{\tau} \to \mathbb{R}$ is a time aggregator that satisfies W(x, u(z)) = u(x) for every $x \in X$.

Observe that for each $t \in \mathcal{T}$ and $h \in \mathcal{F}$, the function $V_t(\cdot, h)$ is \mathcal{G}_t -measurable because h_t is \mathcal{G}_t -measurable and $(I_{t,\omega})_{t,\omega}$ is adapted to \mathcal{G} .

A general recursive representation of $\succeq_{t,\omega}$ can be thus identified by its parameters $(W, u, (I_{t,\omega})_{t,\omega})$. Below I describe the most common types of specifications under the assumptions that $X = [0, \infty)$ and u(0) = 0. Observe that in these examples we have that z = 0 and W(x, u(z)) = u(x) for every $x \in [0, \infty)$.

- Recursive discounted expected utility (RDEU) preferences, where $W(x, y) = u(x) + \beta y$, $\beta \in \mathbb{R}_+$ and $I_{t,\omega}(\xi) = \mathbb{E}_{P_{t,\omega}}\xi$ with each $P_{t,\omega}$ being a probability on $(\Omega, \mathcal{G}_{t+1})$.
- Recursive second-order expected utility preferences, where $W(x, y) = u(x) + \beta y$, $\beta \in \mathbb{R}_+$ and $I_{t,\omega}(\xi) = \phi^{-1}(\mathbb{E}_{P_{t,\omega}}\phi(\xi))$ for some strictly increasing and concave function $\phi : u(X) \to \mathbb{R}$ and each $P_{t,\omega}$ is a probability on Ω . Such a class of recursive preferences offers a simple separation between risk aversion and

intertemporal substitution. The most important instances of such preferences are Epstein–Zin preferences (EZ) Epstein and Zin (1989), which are given by⁵

$$u(x) = \frac{x^{\rho}}{\rho} \quad 0 \neq \rho < 1,$$

and

$$\phi(x) = \frac{\rho}{\alpha} x^{\frac{\alpha}{\rho}} \quad 0 \neq \alpha < 1, 0 \neq \rho < 1.$$

• Recursive maxmin expected utility (RMEU) preferences, see Epstein and Wang (1994), Epstein and Schneider (2003) where $I_{t,\omega}(\xi) = \min_{p \in C_{t,\omega}} \mathbb{E}_p \xi$, with each set $C_{t,\omega}$ being a convex and weak*-compact set of probabilities with full support on $(\Omega, \mathcal{G}_{t+1})$.

3 Relevant domains of consumption

Instead of analyzing all possible consumption programs, dynamic models in economics consider restricted, relevant subsets of consumption programs that are both practically meaningful and mathematically structured. These domains arise from practical constraints and technical requirements.

For example, in consumption-savings applications, consumption c_t at every period t is a non-trivial function of income y_t , and uncertainty about income resolves gradually over time. Programs featuring one-shot resolution of uncertainty are therefore unnecessary for solving this kind of problem. In consumption-based asset pricing models (e.g., see Martin 2013), the main variable of interest is log consumption growth, which is modeled as an i.i.d. process for technical convenience rather than following a general structure.

For this reason, I suggest that the axioms of recursive preferences need to hold only on a strict subset of \mathcal{F} , a *relevant* domain in applications.

Consumption programs resolving gradually. A consumption program $f \in \mathcal{F}$ involves early resolution if for some $t \ge 1$, f_t is measurable w.r.t. \mathcal{G}_{τ} for some $\tau < t$. In words, this means that time *t* consumption is known at the earlier period τ .

Definition 2 (*Domain of gradual resolution*) For every $t \in T$, let \mathcal{F}_t denote the set given by

 $\mathcal{F}_t = \{ f \in \mathcal{F} : f_t \text{ is } \mathcal{G}_\tau \text{-measurable for some } \tau < t \implies f \in \mathcal{D} \}.$

Define the relevant domain to be $GR = \bigcap_{t=1}^{T} \mathcal{F}_t$.

In words, this means that a consumption program in GR either involves no early resolution or it is deterministic. Therefore, it holds that $\mathcal{D} \subseteq GR \subsetneq \mathcal{F}$. To illustrate,

⁵ Epstein and Zin (1989) consider a more general class of certainty equivalents, but for simplicity I focus on the case of expected utility.



Fig. 1 Gradual resolution versus early resolution when $S = \{a_1, a_2\}$ and $\Omega = S^2$

Fig. 1 provides an example of consumption programs (assuming $x \neq y$) that resolve early (bottom) and gradually (top). Therefore, only the top consumption program belongs to *GR*.

Independent consumption programs. Another important example of consumption domain relevant in applications is that of consumption programs that are independently distributed over time. In order to consider a notion of independent programs, one has to consider the same setup as in Strzalecki (2013). Specifically, we have that $\Omega = S^T$ with $T \ge 2$, where (S, Σ) is a finite measurable space. Moreover, $\Sigma = 2^S$ and let $\mathcal{G}_t = \Sigma^t \times \{\emptyset, S\}^{T-t}$. In words, this means that at time *t* one knows the realization of $(\omega, t) = (s_1, \ldots, s_t) := s^t$, but is ignorant about the future. Observe that in this case we have $\mathcal{G}_t((s_1, \ldots, s_T)) = \{s_1\} \times \ldots \times \{s_t\} \times \{\emptyset, S\}^{T-t}$.

Definition 3 The set of independent consumption programs is given by

 $IND = \{h \in \mathcal{F} : \text{ there exist } (f_t)_t \text{ with } f_t \in X^S \text{ such that } h_t(s_1, \dots, s_{t-1}, \cdot) = f_t(\cdot)\}.$

In words, this set contains consumption programs that are "independent" (h_t does not depend on (s_1, \ldots, s_{t-1}) but not necessarily "identically distributed" (h_t depends on a function $f_t : S \to X$ which is not identical over time).

Properties of *GR* **and** *IND*. The consumption domains *GR* and *IND* are also mathematically rich enough to axiomatize a general recursive representation and disentangle a general notion of risk aversion from intertemporal substitution. These results, which I discuss in Sect. 4.1, rely on the following proposition.

Proposition 1 *The consumption domains GR and IND are separable and connected metric spaces.*

4 Recursive utility over the relevant domain

Let \mathcal{R} satisfy $\mathcal{D} \subseteq \mathcal{R} \subseteq \mathcal{F}$. I show that if \mathcal{R} is topologically connected, then one can obtain an axiomatic representation of recursive preferences over \mathcal{R} . The first axiom is a standard continuity requirement.



Axiom 1 (*Continuity*) For every $h \in \mathcal{R}$ the sets

$$\left\{f\in\mathcal{F}:f\succeq_{t,\omega}h\right\},\,$$

and

$$\left\{f\in\mathcal{F}:h\succeq_{t,\omega}f\right\},\,$$

are closed.⁶

Given $t, t' \in T$ such that $t \leq t', x, y, z \in X$, and $d \in D$, $(d_{-t-1}, y, x_{T-t'}, z_{t-t'})$ denotes the deterministic consumption stream that pays d_{τ} at times $\tau = 0, \dots, t-1$, y at time t, x at times $\tau = t+1, \dots, T+t-t'$ and z at times $\tau = T+t-t'+1, \dots, T$.

Axiom 2 (*Stationarity*) There exists $z \in X$ such that for every $t \le t', \omega, \omega' \in \Omega$, $d \in \mathcal{D}, y, \overline{y}, x, \overline{x} \in X$

This axiom of stationarity differs from the standard stationarity axiom because of the finite horizon structure. It requires (i) the existence of a "null element" $z \in X$, which represents a state that yields no utility, and (ii) that preferences over deterministic programs remain invariant under time shifts.

To illustrate an implication of stationarity: if today I prefer consuming x over \bar{x} for T periods and then consuming nothing (z) in the final period, this is equivalent to preferring x over \bar{x} for T periods starting tomorrow. See Fig. 2 for a graphical representation.

The next axiom, which I refer to as (restricted) consequentialism, requires that the decision maker at a node (t, ω) does not care about (i) what an act pays on unrealized events nor (ii) what it pays at earlier time periods.

Axiom 3 (*Restricted Consequentialism*) For all $t \in \mathcal{T}$ and $\omega \in \Omega$, and all acts $f, g \in \mathcal{R}$, if $f_k(\omega') = g_k(\omega')$ for all $k \ge t$ and all $\omega' \in \mathcal{G}_t(\omega)$, then $f \sim_{t,\omega} g$.

Observe that the above axiom implies that the ranking of a program $f \in \mathcal{R}$ by $\succeq_{t,\omega}$ depends only on $(f_t(\omega), f_{t+1}, \ldots, f_T)$.

Finally, the last axiom excludes preference reversals as new information arrives.

 $^{^6}$ Recall that \mathcal{F} is endowed by the product topology, and that therefore \mathcal{R} can be endowed with the relative topology.

Axiom 4 (*Restricted Dynamic Consistency*) For all $t \in \mathcal{T}$, and $\omega \in \Omega$, and programs $f, g \in \mathcal{R}$ that yield identical outcomes up to and including period t, if $f \succeq_{t+1,\omega'} g$ for all $\omega' \in \mathcal{G}_t(\omega)$, then $f \succeq_{t,\omega} g$ and if $f \succ_{t+1,\omega'} g$ for some $\omega' \in \mathcal{G}_t(\omega)$, then $f \succ_{t,\omega} g$.⁷

Observe that these axioms are *restricted* to hold only for the relevant domain of consumption \mathcal{R} , and need not hold on the entire domain \mathcal{F} . The next representation theorem characterizes recursive utility under very general conditions (cf. Kreps and Porteus (1978); Johnsen and Donaldson (1985); Chew and Epstein (1991); Skiadas (1998); Wang (2003); Hayashi (2005); Bommier et al. (2017)), allowing for both changing beliefs and ambiguity sensitive preferences. The only loss of generality is constituted by the exclusion of an infinite horizon, which, however, can be overcome by means of appropriate technical conditions.

Theorem 1 (Recursive representation) Assume that \mathcal{R} is connected and satisfies $\mathcal{D} \subseteq \mathcal{R} \subseteq \mathcal{F}$. Preferences $\succeq_{t,\omega}$ satisfy axioms 1–4 if and only if they admit a recursive representation over \mathcal{R} .

Proof See the appendix.

At a technical level, the main difficulty introduced by weakening the axiom of dynamic consistency is related to showing that $\mathcal{R} \subseteq \mathcal{F}$ is rich enough to construct the certainty equivalents of the recursive representation. Furthermore, a priori, it is not obvious that there are connected domains \mathcal{R} strictly contained within \mathcal{F} . In the appendix (see the proof Proposition 1 and Remark 4), I show that GR and IND are connected. Hence, Theorem 1 shows that gradual resolution of uncertainty is enough to provide a recursive representation of preferences. Specifically, Theorem 1 enables axiomatizations of notable special cases of recursive preferences over a restricted domain, such as Epstein–Zin preferences.

Definition 4 (*Epstein–Zin over a restricted domain*) Say that preferences $\succeq_{t,\omega}$ admit an Epstein–Zin (EZ) representation (α, ρ, β) over $\mathcal{R} \subseteq \mathcal{F}$ if there exist $(V_t(\omega, \cdot))_{t,\omega}$ that represent $\succeq_{t,\omega}$ such that

$$V_t(\omega, h) = \frac{h_t(\omega)^{\rho}}{\rho} + \beta (\mathbb{E}_{P_{t,\omega}}(V_{t+1}(\cdot, h)^{\frac{\alpha}{\rho}})^{\frac{\rho}{\alpha}} \quad \text{for every } h \in \mathcal{R},$$
(3)

where $0 \neq \alpha < 1$ and $0 \neq \rho < 1$.

For simplicity, I do not provide a complete axiomatization of these preferences here. However, such an axiomatization can be obtained by augmenting axioms 1-4 with additional axioms of time separability over deterministic consumption programs and an axiom of homotheticity of preferences.

I close with a few remarks.

⁷ It is possible to consider a weaker axiom of dynamic consistency, which would result in a certainty equivalent that $I_{t,\omega}$ need not be strictly monotone. This could be done by defining appropriately the notion of a $\geq_{t,\omega}$ -nonnull event. Then one can require that if $f \succ_{t+1,\omega'} g$ for every ω' in a $\geq_{t,\omega}$ -nonnull event, then $f \succ_{t,\omega} g$. I chose to present the stronger representation, as DC is easier to state.

Remark 1 One could wonder whether given a recursive representation $(W, u, (I_{t,\omega})_{t,\omega})$ on \mathcal{R} the only "reasonable" extension to \mathcal{F} is given by the straightforward extension of $(W, u, (I_{t,\omega})_{t,\omega})$ to \mathcal{F} . In Sect. 5, I show that one can extend preferences in a different fashion. Specifically, I introduce preferences that have an Epstein–Zin representation on \mathcal{R} but on $\mathcal{F} \setminus \mathcal{R}$ admit the representation introduced by Selden and Stux (1978) and Selden (1978). Notably, such a "hybrid" model features indifference to timing of resolution of uncertainty.

Remark 2 The theorem makes no reference to uniqueness of the representation. Uniqueness can be achieved by adding further conditions that imply uniqueness of $u : X \to \mathbb{R}$. For example, one can assume that X is the set of lotteries over a finite set Z and obtain uniqueness of u by means of specific axioms such as independence.

4.1 Separating intertemporal substitution from attitudes toward uncertainty

A simple yet important implication of Theorem 1 is that to separate risk aversion from the intertemporal rate substitution it is enough to observe only choices over a subset $\mathcal{D} \subseteq \mathcal{R} \subseteq \mathcal{F}$.

Comparative risk aversion can be defined in a similar fashion as in Epstein and Zin (1989, pp. 949–950) and Chew and Epstein (1991, Theorem 3.2). For any $f \in \mathcal{R}$, (t, ω) and $d \in \mathcal{D}$, denote with $(f_t(\omega), d_{T-t})$ the consumption stream that pays $f_t(\omega)$ at time t and d_{τ} at times $\tau = t + 1, ..., T$.

Definition 5 $\geq_{t,\omega}^1$ is more risk averse than $\geq_{t,\omega}^2$ if for every $f \in \mathcal{R}, d \in \mathcal{D}$ and (t, ω) with t < T

$$(f_t(\omega), d_{T-t}) \succeq_{t,\omega}^2 (f_t(\omega), f_{t+1}, \dots, f_T)$$
$$\implies (f_t(\omega), d_{T-t}) \succeq_{t,\omega}^1 (f_t(\omega), f_{t+1}, \dots, f_T),$$

and

$$(f_t(\omega), d_{T-t}) \succ_{t,\omega}^2 (f_t(\omega), f_{t+1}, \dots, f_T)$$
$$\implies (f_t(\omega), d_{T-t}) \succ_{t,\omega}^1 (f_t(\omega), f_{t+1}, \dots, f_T).$$

We obtain the following comparative statics result.

Proposition 2 Consider preferences $\geq_{t,\omega}^{i}$, i = 1, 2 that admit the representation in (2). $\geq_{t,\omega}^{1}$ is more risk averse than $\geq_{t,\omega}^{2}$ if and only if they admit recursive representations $(W^{i}, u^{i}, (I_{t,\omega}^{i})_{t,\omega}), i = 1, 2$ such that $u^{1} = u^{2}, W^{1} = W^{2}$ and $I_{t,\omega}^{1}(\xi) \leq I_{t,\omega}^{2}(\xi)$ for every $\xi \in \{\xi \in B_{0}(\mathcal{G}_{t+1}, V_{t+1}) : \xi = V_{t+1}(\cdot, f), f \in \mathcal{R}\}$ and every (t, ω) .

Remark 3 Observe that if $\mathcal{R} = \mathcal{D}$ then the "only if" part of the statement is trivially true since $I_{t,\omega}^i$ are defined on deterministic prospects so that $I_{t,\omega}^1 = I_{t,\omega}^2$. More in general, this will be true whenever

$$\{\xi \in B_0(\mathcal{G}_{t+1}, V_{t+1}) : \xi = V_{t+1}(\cdot, f), f \in \mathcal{R}\}$$

= $\{\xi \in B_0(\mathcal{G}_{t+1}, V_{t+1}) : \xi = V_{t+1}(d), d \in \mathcal{D}\}.$ (4)

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Now consider the special case of EZ preferences $\succeq_{t,\omega}^{i}$ with representation $(\alpha_i, \rho_i, \beta_i)$, i = 1, 2, over \mathcal{R} such that (4) does not hold. In this case we obtain the following.

Corollary 1 $(\succeq_{t,\omega})_{t,\omega}^1$ is more risk averse than $(\succeq_{t,\omega})_{t,\omega}^2$ if and only if $\beta_1 = \beta_2$, $\rho_1 = \rho_2$ and $\alpha_1 \leq \alpha_2$.

Proof The result follows immediately by the previous proposition upon observing that $I(\xi) = (\mathbb{E}_P \xi^{\alpha})^{\frac{1}{\alpha}}$ is increasing in α (see Theorem 16, Hardy et al. (1952)).

This result establishes that domains of consumption such as $\mathcal{R} = GR$ or $\mathcal{R} = IND$ are rich enough to disentangle risk aversion from intertemporal substitution, since (4) does not hold in these cases.

5 The Epstein-Zin-Selden-Stux model

Theorem 1 provides a foundation for recursive preferences over a restricted domain, including EZ preferences. This naturally raises the question: can EZ preferences, defined over a restricted domain, be meaningfully extended to the broader domain \mathcal{F} ? Here, I introduce a novel class of preferences that admit an EZ representation over a restricted domain and are capable of addressing experimental evidence regarding how individuals value the timing of uncertainty resolution. The new model I introduce "merges" EZ preferences with DOCE preferences.

DOCE preferences. Axiomatized in Selden (1978) and Selden and Stux (1978), DOCE preferences replace risky consumption in each period by certainty equivalents with respect to a utility function $v(\cdot)$ and evaluate the resulting sequence of certainty equivalents with discounted utility with respect to a utility function $u(\cdot)$. Unlike EZ preferences, DOCE preferences are indifferent to the timing of the resolution of uncertainty (see Selden and Stux 1978 and Kubler et al. (2019)). Here I adopt Strzalecki's (2013) approach to define indifference to the timing of resolution of uncertainty. Therefore, I consider the IID setting described in Sect. 3 and assume that $X = [0, \infty)$ or $X = (0, \infty)$.

Definition 6 Preferences $\succeq_{t,\omega}$ admit a DOCE representation if they can be represented over \mathcal{F} by a family $(V_t(s^t, \cdot))_{s^t}$ such that, for some probability measure P on S and continuous, strictly increasing functions $u, v : X \to \mathbb{R}$, the following holds for every $h \in \mathcal{F}$:

$$V_t(s^t, h) = u(h_t(s^t)) + \sum_{j=1}^{T-t} \beta^j u(v^{-1} \left[\mathbb{E}_{\prod_{\tau=1}^j P(s_{t+\tau})} (v(h_{t+j}(s^t, \cdot))) \right] \right)$$

Of particular interest is the positively homogeneous case where, for $0 \neq \alpha < 1$ and $0 \neq \rho < 1$, we have $u(x) = \frac{x^{\rho}}{\rho}$ and $v(x) = \frac{x^{\alpha}}{\alpha}$. As in Epstein–Zin preferences, this specification enables a separation between the EIS, given by $\frac{1}{1-\rho}$, and risk aversion,

given by $1 - \alpha$. Hall (1985), Zin (1987), Attanasio and Weber (1989), and Kubler et al. (2019) have studied applications of different variations of such preferences.

While EZ preferences permit only a partial separation between the EIS and relative risk aversion, DOCE preferences achieve a complete separation.⁸ Unlike EZ preferences, DOCE preferences may fail to satisfy dynamic consistency. Notably, however, DOCE preferences can be dynamically consistent on certain restricted domains. For instance, as suggested by Kubler et al. (2019), the homogeneous special case of DOCE preferences is dynamically consistent when restricted to $\mathcal{R} = IND$. In this scenario, DOCE preferences become ordinally equivalent to EZ preferences over the domain $\mathcal{R} = IND$.

EZSS Preferences. The Epstein–Zin–Selden–Stux (EZSS) preferences introduced here are defined by Epstein–Zin utility on the relevant domain and by the DOCE model elsewhere.

Definition 7 Preferences $\succeq_{t,\omega}$ admit an EZSS representation $(\alpha, \rho, \beta, \mathcal{R})$ if they can be represented over \mathcal{F} by a family $(V_t(s^t, \cdot))_{s^t}$ such that, for some probability measure P on S, and parameters $0 \neq \alpha < 1, 0 \neq \rho < 1$, the following holds:

For every $h \in \mathcal{F} \setminus \mathcal{R}$:

$$V_t(s^t, h) = \frac{h_t(s^t)^{\rho}}{\rho} + \sum_{j=1}^{T-t} \beta^j \frac{1}{\rho} \left[\mathbb{E}_{\prod_{\tau=1}^j P(s_{t+\tau})} h_{t+j}^{\alpha}(s^t, \cdot) \right]^{\frac{\rho}{\alpha}},$$
(5)

And for every $h \in \mathcal{R}$:

$$V_t(s^t, h) = \frac{h_t(s^t)^{\rho}}{\rho} + \beta \left(\mathbb{E}_P \left[Vt + 1(\cdot, h)^{\frac{\alpha}{\rho}} \right] \right)^{\frac{\rho}{\alpha}}.$$
 (6)

Therefore, preferences that admit an EZSS representation are guaranteed to satisfy dynamic consistency only on the relevant domain \mathcal{R} . At the same time, as I am going to show, they satisfy indifference to the timing of resolution of uncertainty "outside" of \mathcal{R} . Moreover, thanks to Proposition 2, EZSS preferences can separate risk aversion from intertemporal substitution. Finally, as noted previously, when the relevant domain is $\mathcal{R} = IND$, EZ preferences are ordinally equivalent to DOCE preferences, making EZSS preferences also ordinally equivalent to DOCE preferences in this special case.

Axiomatic foundation. Consider preferences $\succeq_{t,\omega}$ that admit an EZ representation (α, ρ, β) over $\mathcal{R} \subseteq \mathcal{F}$. Since we assume that $\succeq_{t,\omega}$ admit an EZ representation, this implicitly assumes that the preferences $\succeq_{t,\omega}$ satisfy Axioms 1–4.

The first axiom requires consequentialism over the entire domain \mathcal{F} .

Axiom 5 (*Consequentialism*) For all $t \in \mathcal{T}$ and $\omega \in \Omega$, and all acts $f, g \in \mathcal{F}$, if $f_k(\omega') = g_k(\omega')$ for all $k \ge t$ and all $\omega' \in \mathcal{G}_t(\omega)$, then $f \sim_{t,\omega} g$.

⁸ Epstein and Zin (1989) observe that attitudes toward timing are intertwined with the EIS and risk aversion. In contrast, DOCE preferences are indifferent toward timing.

Now observe that because in this setting the state space has a product structure, it is possible to rank consumption programs in terms of temporal resolution of uncertainty (see Strzalecki 2013).

Definition 8 Fix $t \leq T - 2$. Say that $h \in \mathcal{F}$ resolves earlier than $h' \in \mathcal{F}$ whenever there exist $f_{t+2}, \ldots, f_T \in X^S$ and $x_0, \ldots, x_{t+1} \in X$ such that $h_j = h'_j = x_j$ for all $j \leq t+1, h_j(s_1, \ldots, s_j) = f_j(s_{t+1})$ for $j \geq t+2$, and $h'_j(s_1, \ldots, s_j) = f_j(s_{t+2})$.

The next axiom requires indifference toward the timing of resolution of uncertainty on the domain $\mathcal{F} \setminus \mathcal{R}$.

Axiom 6 (*Indifference to timing*) Consider $h, h' \in \mathcal{F} \setminus \mathcal{R}$ such that for some $\bar{s}^t = (\bar{s}_1, \ldots, \bar{s}_t)$ with $1 \le t \le T - 2$ the consumption program

$$(h_0, h_1(\bar{s}_1), h_2(\bar{s}_1, \bar{s}_2), \dots, h_t(\bar{s}^t), \dots, h_T(\bar{s}^t, \cdot)),$$

resolves earlier than

$$(h'_0, h'_1(\bar{s}_1), h'_2(\bar{s}_1, \bar{s}_2), \dots, h'_t(\bar{s}^t), \dots, h'_T(\bar{s}^t, \cdot))$$

and $h_{\tau}(s_1, \ldots, s_{\tau}) = h'_{\tau}(s_1, \ldots, s_{\tau})$ for every (s_1, \ldots, s_{τ}) with $\tau < t$ and $h_{\tau}(s_1, \ldots, s_{\tau}) = h'_{\tau}(s_1, \ldots, s_{\tau})$ for every (s_1, \ldots, s_{τ}) such that $\tau \ge t$ and $(s_1, \ldots, s_t) \ne (\bar{s}_1, \ldots, \bar{s}_t)$. Then it holds that $h \sim_{t',\omega} h'$ for every $t' \le t$.

In words, if h resolves earlier than h', but otherwise these programs are equivalent, one should be indifferent.

Consider now preferences $\succeq_{t,\omega}^{EZ}$ that have an EZ representation (α, ρ, β) on \mathcal{F} . The next axiom requires that, for each period, risky consumption profiles are assessed through certainty equivalents, consistent with such Epstein–Zin preferences $\succeq_{t,\omega}^{EZ}$.

Axiom 7 (*Consistency with Epstein–Zin*) Let $h, h' \in \mathcal{F} \setminus \mathcal{R}$ be such that there exist, $t, f, f' : S \to X$ and (s_1, \ldots, s_{t-1}) such that $h_t(s_1, \ldots, s_{t-1}, \cdot) = f(\cdot)$, $h'_t(s_1, \ldots, s_{t-1}, \cdot) = f'(\cdot), h_\tau = h'_\tau = 0$ for all $\tau \neq t$ and $h_t(\bar{s}^t) = h'_\tau(\bar{s}^t) = 0$ whenever $(\bar{s}_1, \ldots, \bar{s}_{t-1}) \neq (s_1, \ldots, s_{t-1})$. Then

$$h \succeq_{t,\omega} h' \iff h \succeq_{t,\omega}^{EZ} h'.$$

The last axiom is taken from Selden and Stux (1978) (see their Assumption 3).

Axiom 8 (*Risk Independence*) Given any pair $h, h' \in \mathcal{F} \setminus \mathcal{R}$ which are identical except at the node (s_1, \ldots, s_{t-1}) , then letting $\bar{h}_t(s_1 \ldots, s_{t-1}, \cdot) = h_t(s_1 \ldots, s_{t-1}, \cdot)$, $\bar{h}'_t(s_1 \ldots, s_{t-1}, \cdot) = h'_t(s_1 \ldots, s_{t-1}, \cdot)$, and $\bar{h}_t = \bar{h}'_t = 0$ otherwise,

$$\bar{h} \sim_{t,\omega} \bar{h}' \implies h \sim_{t,\omega} h'.$$

These axioms characterize EZSS preferences.

Theorem 2 Assume that \mathcal{R} is connected and satisfies $\mathcal{D} \subseteq \mathcal{R} \subseteq \mathcal{F}$. Preferences $\succeq_{t,\omega}$ satisfy axioms 5–8 if and only if they admit an EZSS representation $(\alpha, \rho, \beta, \mathcal{R})$.

By Theorem 1, we can consider Epstein–Zin (EZ) preferences over a restricted domain. Theorem 2 shows that these preferences can be extended to an EZSS representation. However, note that the continuity axiom does not hold over the entire domain \mathcal{F} . As a result, the EZSS model may exhibit discontinuous jumps in utility when transitioning from the domain \mathcal{R} to $\mathcal{F}\setminus\mathcal{R}$. Nevertheless, as I will show in Sect. 5.1.1, these potential discontinuities do not hinder the application of the EZSS model to relevant applied problems, such as consumption-investment decisions.

At the same time, such an extension satisfies a form of indifference to the timing of resolution of uncertainty. Clearly, the full extent of such an indifference to timing of resolution depends on how "large" or "small" \mathcal{R} is (when $\mathcal{R} = \emptyset$, the model collapses to DOCE preferences, exhibiting full indifference to the timing of resolution). This flexibility can lead to interesting applications, as I explore next.

5.1 Application: timing premia

The notion of a timing premium was introduced by Epstein et al. (2014). It quantifies how much an individual would pay to resolve all uncertainty at t = 1, as opposed to experiencing a gradual resolution of uncertainty over time. This concept has also been applied to dynamic models of retirement and social security (Caliendo et al. 2023), where it is used to measure the quantitative impact of uncertainty regarding the timing of retirement. Understanding the determinants of the timing premium is therefore crucial for evaluating the financial costs of policies that influence the retirement age.

For Epstein–Zin recursive utility, $\alpha < \rho$ implies a positive timing premium, while $\alpha > \rho$ implies a negative timing premium. In contrast, DOCE preferences always imply a zero timing premium (Kubler et al. 2019). Epstein et al. (2014) argue that standard parameter assumptions in the Epstein–Zin model imply that consumers would pay implausibly high timing premia for early resolution of consumption risk. However, Meissner and Pfeiffer (2022) find that, on average, the timing premium is moderate, around 5%, with about 40% of subjects showing indifference to the timing of uncertainty resolution. Moreover, the estimated preference parameters satisfy

relative risk aversion $< \text{EIS}^{-1}$ or equivalently $\alpha > \rho$,

a relationship that, under Epstein–Zin preferences, predicts a negative timing premium—contradicting these experimental findings.

I show that, in contrast, the EZSS model can explain this evidence. Specifically, when the relevant domain is *IND*, EZSS preferences are compatible with a *positive* timing premium even if $\alpha > \rho$. With the estimated parameters (α , ρ) from Meissner and Pfeiffer (2022), the timing premium under EZSS preferences is approximately 4%, indicating a moderate preference for early resolution of uncertainty. This level is consistent with the experimental finding in Meissner and Pfeiffer (2022) of a timing premium around 5%. Hence, EZSS model provides an additional feature that neither Epstein–Zin nor DOCE preferences can explain individually. Furthermore, when the



relevant domain is GR, the timing premium is zero. Finally, I show applications of this result to a simple consumption-investment problem over a finite horizon.

Timing premium of EZSS preferences. Consider again the IID setting, and as in Meissner and Pfeiffer (2022) assume that T = 2. Let preferences $\succeq_{t,\omega}$ admit an EZSS representation (α , ρ , β , \mathcal{R}) with corresponding utilities $(V_t(s^t, \cdot))_{s^t}$. To introduce the notion of a timing premium, I use the ordinally equivalent representation defined by

$$\hat{V}_t = (\rho V_t)^{\frac{1}{\rho}},$$

for t = 0, 1, 2.⁹ For each $h = (h_0, h_1, h_2) \in IND$, let \bar{h} denote the corresponding early resolution consumption program, defined as $\bar{h} = (h_0, h_1, \bar{h}_2)$, where $\bar{h}_2(s_1, s_2) = h_2(s_1)$ for every $(s_1, s_2) \in S^2$. Following Epstein et al. (2014), I define the timing premium as follows.

Definition 9 For every $h \in IND$, the timing premium is defined as

$$\pi^*(h) = 1 - \frac{\hat{V}_0(\bar{h})}{\hat{V}_0(h)}$$

Next, I analyze specific values of the timing premium for EZSS preferences, using the estimated preference parameters from Meissner and Pfeiffer (2022).

Example 1 Figure 3 shows the example taken from the experiment in Meissner and Pfeiffer (2022), with consumption program h and its corresponding early resolution program \bar{h} . Here, $S = \{a_1, a_2\}$, $h = (0, 100, h_2)$, where $h_2(s_1, s_2) = f(s_2)$, $f : \{a_1, a_2\} \rightarrow [0, \infty)$, $f(a_1) = 170$, and $f(a_2) = 10$. Consider EZSS preferences with representation $(\alpha, \rho, \beta, \mathcal{R})$, based on the estimated parameters from Table 4 in Meissner and Pfeiffer (2022), so that $(\alpha, \rho, \beta) = (0.819, 0.579, 1.193)$.¹⁰

 $^{^{9}}$ With this normalization, recursive utilities are positively homogeneous, which simplifies the definition of the timing premium.

¹⁰ Note that here $\beta > 1$, which Meissner and Pfeiffer (2022) discuss as a potentially counter-intuitive result.

- Case 1: $\mathcal{R} = \mathcal{F}$: When $\mathcal{R} = \mathcal{F}$, the EZSS model reduces to standard Epstein–Zin preferences. Since $\alpha > \rho$, the timing premium is negative (Epstein et al. 2014), reflecting a moderate preference for late resolution of uncertainty. Specifically, I obtain $\pi^*(h) \approx -4.2\%$.
- Case 2: $\mathcal{R} = IND$: When $\mathcal{R} = IND$, $h \in IND$, but $\bar{h} \notin IND$. In this case, $\pi^*(h) \approx 4\%$, consistent with the average timing premium of approximately 5% reported in Meissner and Pfeiffer (2022).
- Case 3: $\mathcal{R} = GR$: When $\mathcal{R} = GR$, $h, \bar{h} \notin GR$. By Axiom 6, this implies $\pi^*(h) = 0$.

For related calculations, see Sect. 1 in the Appendix.

The main finding in Meissner and Pfeiffer (2022) is a negative correlation between the theoretically predicted timing premia and the timing premia elicited in the experiment. While there is evidence of preferences over the temporal resolution of consumption uncertainty, EZ preferences fail to explain the underlying mechanism. My results indicate that this evidence can be explained by relaxing dynamic consistency while still maintaining the tractability of dynamic programming in applications.

5.1.1 The timing premium in consumption-portfolio decisions

I now examine the implications of the previous analysis of timing premia for dynamic consumption-portfolio decision problems. The main result is that because EZSS preferences can exhibit a positive timing premium even when $\alpha > \rho$, then it can happen that the optimal consumption plan resolves early at the initial time period, while for EZ utility with $\alpha > \rho$ the optimal consumption plan resolves late.¹¹ Finally, for DOCE preferences, the optimal consumption plan resolves gradually over time.

I consider the consumption-portoflio selection problem analyzed in Kubler et al. (2019). Formally, there is a finite horizon of T > 0 periods, where again we consider the IID setting with $X = [0, \infty)$. At each node s^t , the consumer has to decide the level of consumption $c(s_t) \ge 0$. I denote an arbitrary consumption program with $c = (c_0, \ldots, c_T) \in \mathcal{F}$. Moreover, at each node s^t there are J > 0 assets with one period maturities whose returns are described by the vector $R(s^{t+1}) = (R(s^{t+1}))_{j=1}^J$, and $n_j(s^t)$ denotes the amount invested in asset j at that node. Therefore, an arbitrary portfolio investment strategy $n = (n_0, \ldots, n_{T-1})$ is a process such that each $n_t : S^t \to \mathbb{R}^J$ is \mathcal{G}_t -measurable. Denote with \mathcal{N} the set of all investment strategies.

The consumer begins with initial wealth w. The budget constraints are defined as follows. In period 1, the consumer allocates income between consumption and asset purchases:

$$c(s_0) = w - \sum_{j=1}^{J} n_j(s_0)$$

 \triangle

¹¹ I thank an anonymous referee for suggesting to consider this application.

For periods $1 \le t < T$, at each node s^t , the income is given by the return on the portfolio chosen in the previous period, and the budget constraint is

$$c(s^{t}) = n(s^{t-1}) \cdot R(s^{t}) - \sum_{j=1}^{J} n_{j}(s^{t}),$$

where $n(s_{t-1}) \cdot R(s_t)$ denotes the dot product between the asset holdings from period t-1 and the return vector at node s_t . In the final period T, no new asset purchase is made so that the constraint becomes

$$c\left(s^{T}\right) = n\left(s^{T-1}\right) \cdot R\left(s^{T}\right).$$

Finally, preferences over \mathcal{F} are represented by the relation $\succeq_{t,\omega}$, which admits an EZSS representation $(\alpha, \rho, \beta, \mathcal{R})$ with corresponding utilities $(V_t(s^t, \cdot))_{s^t}$. Consequently, the resolute consumer's problem (ex-ante optimal; see, e.g., McClennen (1990)) involves choosing a consumption plan and asset holdings across all nodes to maximize utility:

$$\max_{\substack{(c,n)\in\mathcal{F}\times\mathcal{N}\\ \text{subject to } c \ge 0,}} V_0(c)$$

$$\operatorname{subject to } c \ge 0,$$

$$c(s_0) = w - \sum_{j=1}^J n_j(s_0),$$

$$c(s^t) = n(s^{t-1}) \cdot R(s^t) - \sum_{j=1}^J n_j(s^t), \quad t = 1, \dots, T-1,$$

$$c\left(s^T\right) = n\left(s^{T-1}\right) \cdot R\left(s^T\right).$$

To examine the implications of the previous analysis on timing premia, I now consider a special three-period example with two states and two assets, one of which is riskless. Unlike EZ and DOCE preferences, under the condition $\alpha > \rho$, the positive timing premium associated with EZSS utility induces the agent to avoid investing in the risky asset at the second-period nodes, leading to early resolution of the optimal consumption plan.

Example 2 Let T = 2 and $S = \{H, L\}$, and assume the consumer can invest in a riskless asset f and a risky asset r in each period. I compute the optimal resolute (ex-ante optimal) consumption and investment plans under EZ, DOCE, and EZSS preferences, using parameter estimates from Meissner and Pfeiffer (2022), so that $\alpha = 0.819 > \rho = 0.579$ and $\beta = 1.193$.

Assume further that w = 10 and $R_f(s^t) = 1$ for every s^t . The stochastic returns on the risky asset are given by:

$$R_r(H) = R_r(L, H) = R_r(H, H) = 1.1, \quad R_r(L) = R_r(L, L) = R_r(H, L) = 0.9.$$



Fig. 4 Trees representing optimal consumption for EZ (top-left), DOCE (top-right), and EZSS (bottomcenter) preferences when $\alpha > \rho$. EZ resolves late, DOCE resolves gradually, while EZSS resolves early

The two states are equally likely, with $P(H) = P(L) = \frac{1}{2}$. Then the optimal investment strategies are as follows:

1. When $\mathcal{R} = \mathcal{F}$, the preferences reduce to Epstein–Zin preferences. In this case, the optimal investment strategy is:

$$n_r(s_0) = 0$$
, $n_f(s_0) = 4.02$, $n_r(H) = n_r(L) = 0.76$, $n_f(H) = n_f(L) = 1.66$.

2. When $\mathcal{R} = GR$, the optimal investment strategy is:

$$n_r(s_0) = 7.94, \quad n_f(s_0) = 0, \quad n_r(H) = n_r(L) = 0, \quad n_f(H) = n_f(L) = 4.89.$$

3. When $\mathcal{R} = \emptyset$, the preferences reduce to DOCE preferences. In this case, the optimal investment strategy is:

$$n_r(s_0) = 7.95, \quad n_f(s_0) = 0, \quad n_r(H) = n_r(L) = 4.89, \quad n_f(H) = n_f(L) = 0.$$

For related calculations, see Sect. 1 in the Appendix. Figure 4 provides a graphical representation of the optimal consumption plans implied by these optimal investment strategies. Notably, under EZSS preferences, the positive timing premium induces the agent to avoid investing in the risky asset at second-period nodes. Consequently, the consumption program is characterized by early resolution.

In contrast, under EZ preferences, since there is a negative timing premium, it encourages investment in the risky asset exclusively during the second period, resulting in optimal consumption plans that exhibit late resolution.

Finally, with DOCE preferences, the timing premium equals zero. Therefore, the agent invests progressively in the risky asset over multiple periods, leading to gradual resolution of optimal consumption. \triangle

6 Concluding remarks

Models of recursive preferences play a central role in many applications in economics. However, they require strong assumption on behavior. According to one objection against the axiom of dynamic consistency, it is unrealistic to assume the axiom can hold even when the decision maker is confronted with unrealistic choice situations. I provided an axiomatization of recursive preferences based on much weaker assumptions than what is usually assumed. Additionally, I propose a novel parametric model—the Epstein–Zin–Selden–Stux (EZSS) preferences—which generalizes Epstein–Zin preferences. These new preferences explain recent empirical findings concerning individuals' preferences for the timing of resolution of uncertainty. Moreover, they separate risk aversion from intertemporal substitution, while still employing conventional dynamic programming methods. I apply these preferences to a consumption-investment problem and show that—for reasonable parameter values—these preferences lead to investment in risky assets exclusively during the initial period and discourage investment afterward. This result opens promising avenues for future research, especially in retirement planning models.

Appendix

Proof of Proposition 1

Proof First observe that \mathcal{F} is separable and therefore GR is separable since any subset of a separable metric space is separable. I now show that GR is path-connected. Take $h \in GR$ and $d \in \mathcal{D}$. Clearly if $h \in \mathcal{D}$ then the result follows by convexity of \mathcal{D} (recall that X is convex). Assume $h \in GR \setminus \mathcal{D}$. Let $\{P_1^t, \ldots, P_{n^t}^t\}$ denote the partition of Ω that generates \mathcal{G}_t . I construct a continuous path $\iota : [0, 1] \to GR$ that connects h to d. Since X is convex, for every t we just let $\iota_t(\alpha) = (1 - \alpha)h_t + \alpha d_t$. Fix $t \ge 1$. Without loss of generality, assume that $n^t - n^{t-1} = 1$ and $P_{n^{t-1}}^{t-1} = P_{n^t}^t \cup P_{n^t-1}^t$. Consider $\omega, \omega' \in \Omega$ such that $\omega \ne \omega', \omega \in P_{n^t}^t$, and $\omega' \in P_{n^t-1}^t$. If $(1 - \alpha)h_t(\omega) + \alpha d_t =$ $(1 - \alpha)h_t(\omega') + \alpha d_t$, we obtain a contradiction since $h_t(\omega) = h_t(\omega')$ but $h \in GR \setminus \mathcal{D}$. Therefore, $\iota_t(\alpha) \in GR$ for every α . It follows that we can connect via a path any $f \in GR$ to $d \in \mathcal{D}$. Hence, we can connect any $h, h' \in GR$ by a path. We conclude that GR is path-connected and therefore connected.

Remark 4 The set

 $IND = \{h \in \mathcal{F} : \text{ there exist } (f_t)_t \text{ with } f_t \in X^S \text{ such that } h_t(s_1, \dots, s_{t-1}, \cdot) = f_t(\cdot)\},\$

is easily seen to be connected by analogous arguments. Observe that such a domain is the natural extension to T periods of "certain × uncertain" consumption plans [e.g., see Selden (1978), Johnsen and Donaldson (1985)]. Indeed, the two coincide when T = 1.

Proof of Theorem 1

Proof of Theorem 1 I first prove sufficiency of the axioms. First by the axioms of continuity and (restricted) consequentialism, and since \mathcal{R} is connected and separable (the consumption set is separable), Theorem 1 in Debreu (1954) implies that there exist (sequentially) continuous functions $(V_t(\omega, \cdot))_{t,\omega}$ such that

$$V_t(\omega, h) = V_t(h_t(\omega), h_{t+1}, \dots, h_T)$$
 for every $h \in \mathcal{R}$.

Observe that by stationarity there exists a (sequentially) continuous function $u : X \to \mathbb{R}$ such that $V_T(\omega, h) = u(h_T(\omega))$ and $V_t(\omega, (x, z_{T-t})) = u(x)$ for every $\omega \in \Omega$ and t < T.¹² Indeed, stationarity implies that

$$(x, z_{T-t}) \succeq_{t,\omega} (y, z_{T-t}) \iff (x, z_{T-t'}) \succeq_{t',\omega'} (y, z_{T-t'}),$$

so that the functions $(V_t(\omega, \cdot))_{t,\omega}$ can be chosen in such a way that the desired normalization holds. Moreover, by the stationarity axiom we can further normalize $u(\cdot)$ so that u(z) = 0.

I construct $I_{t,\omega}$: $B_0(\mathcal{G}_{t+1}, V_{t+1}(\omega, \mathcal{R})) \to \mathbb{R}$ as follows: for every *h*, by continuity, dynamic consistency, (restricted) consequentialism, and stationarity we can construct $d_{\omega,t} = (d_{t+1}, \ldots, d_T) \in X^{T-t}$ such that for any $\overline{d} \in \mathcal{D}$

$$h \sim_{t,\omega} (\bar{d}_{-t-1}, h_t(\omega), d_{\omega,t}) \in \mathcal{D}.$$
(7)

Observe that all acts in (7) belong to \mathcal{R} . In particular, $d_{\omega,t}$ can be constructed recursively as follows. Starting from t = T - 1, observe that for any $\omega \in \Omega$, there exist $x, y \in X$ such that

$$V_{T-1}(h_{T-1}(\omega), x) \ge V_{T-1}(h_{T-1}(\omega), h_T) \ge V_{T-1}(h_{T-1}(\omega), y).$$

To see this, let $x = h_T(\bar{\omega})$ and $y = h_T(\bar{\omega})$, where $\bar{\omega} = \arg \max_{\omega} u(h_T(\omega))$ and $\omega = \arg \min_{\omega} u(h_T(\omega))$. The statement follows by applying dynamic consistency. Therefore, by continuity and connectedness of X we can find $d_{T-1,\omega} \in X$ such that $h \sim_{T-1,\omega} (\bar{d}_{-t-1}, h_T(\omega), d_{T-1,\omega})$. Now for any t < T - 1 and ω , assume one

¹² Recall that (x, z_{T-t}) denotes the deterministic consumption program that pays x at time t and z in the subsequent periods.

has constructed $d_{t+1,\omega'}$ for every $\omega' \in \mathcal{G}_t(\omega)$. Let $\overline{d}_{t,\omega} = (h_{t+1}(\overline{\omega}), d_{t+1,\overline{\omega}})$ and $\underline{d}_{t,\omega} = (h_{t+1}(\underline{\omega}), d_{t+1,\overline{\omega}})$ where

$$\bar{\omega} = \arg\max_{\omega'} V_{t+1}(h_{t+1}(\omega'), d_{t+1,\omega'}),$$

and

$$\underline{\omega} = \arg\min_{\omega'} V_{t+1}(h_{t+1}(\omega'), d_{t+1,\omega'}),$$

Then by dynamic consistency and stationarity we have

$$V_t(h_t(\omega), d_{t,\omega}) \ge V_t(\omega, h) \ge V_t(h_t(\omega), \underline{d}_{t,\omega}).$$

If any of the two previous inequalities is an equality we are done. Assume both are strict and that there is no $d_{t,\omega}$ such that (7) is satisfied. Then we obtain that

$$\mathcal{D} = \left\{ d \in \mathcal{D} : h \succ_{t,\omega} d \right\} \cup \left\{ d \in \mathcal{D} : d \succ_{t,\omega} h \right\}.$$

By the axiom of continuity we therefore conclude that \mathcal{D} is the union of disjoint open sets, which contradicts the connectedness of *X*. Therefore, there must be $d_{t,\omega}$ such that (7) is verified.

Now observe that by stationarity, the previous result implies that for each (t, ω) , t = 0, ..., T and $\omega, \omega' \in \Omega$ we have $V_t(\omega, \mathcal{R}) = V_t(\omega', \mathcal{R}) \equiv V_t$ (further observe that $V_t \subseteq V_{t'}$ whenever $t' \leq t$). Define

$$I_{t,\omega}: B_0(\mathcal{G}_{t+1}, V_{t+1}) \to \mathbb{R},$$

by $I_{t,\omega}(\xi) = V_{t+1}(d_{\omega,t})$ and where $\xi(\omega) = V_{t+1}(\omega, h)$. Observe that $I_{t,\omega}$ is well defined by dynamic consistency.

I now claim that $I_{t,\omega}$ is strictly monotone, normalized, and continuous. The fact that $I_{t,\omega}$ is normalized follows by definition. Strict monotonicity follows from dynamic consistency. To prove continuity, assume that $\xi_n \to \xi$. Let h_n and h satisfy $\xi_n = V_{t+1}(\cdot, h_n), \xi = V_{t+1}(\cdot, h)$, and $\lim h_n = h$. For a contradiction, suppose that $I_{t,\omega}(\xi_n) \twoheadrightarrow I_{t,\omega}(\xi)$. It follows that $V_{t+1}(d_{t,\omega}^n) \twoheadrightarrow V_{t+1}(d_{t,\omega})$. Hence, there exists $\varepsilon > 0$ such that, for every $N \in \mathbb{N}$, there is an $n \ge N$ with

$$|V_{t+1}(d_{t,\omega}) - V_{t+1}(d_{t,\omega}^n)| \ge \varepsilon > 0.$$

By dynamic consistency it follows that there exists $\epsilon > 0$ such that for every $N \in \mathbb{N}$

$$|V_t(h_t(\omega), V_{t+1}(d_t^n \omega)) - V_t(h_t(\omega), V_{t+1}(d_{t,\omega}))| \ge \epsilon > 0,$$

v for some $n \ge N$. Observe that by continuity we have $V_t(\omega, h_n) \to V_t(\omega, h)$. Hence, we have arrived at a contradiction. Therefore $I_{t,\omega}(\xi_n) \to I_{t,\omega}(\xi_n)$ as desired.

Now assume that $h_t(\omega) = h'_t(\omega)$ and $I_{t,\omega}(V_{t+1}(\cdot, h)) = I_{t,\omega}(V_{t+1}(\cdot, h'))$. By dynamic consistency, it follows that $h \sim_{t,\omega} h'$. Moreover, if $I_{t,\omega}(V_{t+1}(\cdot, h)) >$

 $I_{t,\omega}(V_{t+1}(\cdot, h'))$ then $h_{t,\omega} \succ_{t,\omega} h'$. By Lemma 1 in Gorman (1968) it follows that there exists a continuous function $W_t : X \times V_{t+1} \to \mathbb{R}$ strictly increasing in its second argument such that

$$V_t(\omega, h) = W_t\left(h_t(\omega), I_{t,\omega}\left(V_{t+1}(\cdot, h)\right)\right).$$

Finally observe that by stationarity it holds that $W_t(x, y) = W_{t'}(x, y)$ for every t, t', $x \in X$ and $y \in V_{\max\{t,t'\}+1}$. Therefore, we can set $W \equiv W_0$, which delivers the representation.

I now turn to the necessity of the axioms. It is immediate possible to check that the recursive representation satisfies axiom 3. To show that the representation satisfies continuity, take $h \in \mathcal{R}$ and a sequence $(f_n)_n$ in \mathcal{R} such that $f_n \succeq_{t,\omega} h$ and $\lim f_n = f$. This means that $V_t(\omega, f_n) \ge V_t(h)$ for every *n* so that by sequential continuity of $V_t(\omega, \cdot)$ we obtain that the set

$$\left\{f\in\mathcal{R}:f\succeq_{t,\omega}h\right\},\,$$

is closed. Showing that the set

$$\left\{f\in\mathcal{R}:h\succeq_{t,\omega}f\right\},\,$$

is closed can be done in the same way. Turn now to axiom 2. Let $z \in X$ be such that u(z) = 0 and W(x, u(z)) = u(x) (we know that such a *z* exists by assumption). Now for every $t \le t', \omega, \omega', d \in \mathcal{D}$, $y, \bar{y}, x, \bar{x} \in X$ it holds that $V_{t'+1}(x_{T-t'}) = V_{t+1}((x_{T-t'}, z_{t-t'}))$. It follows that

$$V_{t}(\omega, (d_{-t-1}, y, x_{T-t'}, z_{t-t'})) = W(y, V_{t+1}(x_{T-t'}))$$

$$\geq V_{t}\left(\omega, (d_{-t-1}, \bar{y}, \bar{x}_{T-t'}, z_{t-t'})\right) = W(\bar{y}, V_{t+1}(\bar{x}_{T-t'}))$$

$$\iff V_{t'}(\omega, (d_{-t'-1}, y, x_{T-t'})) = W(y, V_{t'+1}(x_{T-t'}))$$

$$\geq V_{t'}\left(\omega, (d_{-t'-1}, \bar{y}, \bar{x}_{T-t'})\right) = W(\bar{y}, V_{t'+1}(\bar{x}_{T-t'})),$$

which implies that axiom 2 is satisfied. Finally, take $h, h' \in \mathcal{R}$ and (t, ω) with $h_t(\omega) = h'_t(\omega)$. If $h \succeq_{t+1,\omega'} h'$ for every $\omega' \in \mathcal{G}_t(\omega)$ then $V_{t+1}(\omega', h) \succeq_{t+1,\omega'} V_{t+1}(\omega', h')$ which by monotonicity of $I_{t,\omega}$ implies $I_{t,\omega}(V_{t+1}(\cdot, h)) \ge I_{t,\omega}(V_{t+1}(\cdot, h'))$. Since Wis strictly increasing in its second variable, it follows that $V_t(\omega, h) \succeq_{t,\omega} V_t(\omega, h')$ as desired. Moreover, if for some $\omega' \in \mathcal{G}_t(\omega)$ the inequality is strict, then by strict monotonicity of $I_{t,\omega}$ we get $I_{t,\omega}(V_{t+1}(\cdot, h)) > I_{t,\omega}(V_{t+1}(\cdot, h'))$ as desired. \Box

Proof of Proposition 2

Proof First observe that if $W_1 = W_2$, then

$$(f_t(\omega), d_{T-t}) \succeq_{t,\omega}^i (f_t(\omega), f_{t+1}, \dots, f_T) \iff V_{t+1}(d) \ge I_{t,\omega}^i(V_{t+1}^i(\cdot, f)),$$
(8)

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Now if $\succeq_{t,\omega}^1$ is more risk averse than $\succeq_{t,\omega}^2$ then it is straightforward to check that they rank prospects in \mathcal{D} in the same way. It follows that they must admit recursive representations $(W^i, u^i, (I_{t,\omega}^i)_{t,\omega}), i = 1, 2$ such that $u^1 = u^2$ and $W^1 = W^2$. By (8) it follows that $I_{t,\omega}^1(\xi) \leq I_{t,\omega}^2(\xi)$ for every $\xi \in \{\xi \in B_0(\mathcal{G}_{t+1}, V_{t+1}) : \xi = V_{t+1}(\cdot, f), f \in \mathcal{R}\}$. The converse follows immediately by (8).

Proof of Theorem 2

Proof of Proposition 2 First observe that preferences $\geq_{t,\omega}$ satisfy (6) by assumption since they admit an EZ representation over \mathcal{R} . The remainder of the proof uses arguments from Selden and Stux (1978) (proof of Lemma 1). I prove sufficiency of the axioms in the case of \geq_0 , and using consequentialism the result follows for $\geq_{t,\omega}$ analogously. I claim that for every $h \in \mathcal{F} \setminus \mathcal{R}$, there exists $\bar{c} = (h_0, c_1, \ldots, c_T) \in \mathcal{D}$ such that $\bar{c} \sim_0 h$ and

$$c_t = \left[\mathbb{E}_{\prod_{\tau=1}^t P(s_\tau)} h_t^{\alpha} \right]^{\frac{1}{\alpha}},$$

which establishes the representation since \geq_0 has an EZ representation so that

$$V_0((h_0, c_1, \dots, c_T))) = u(h_0) + \sum_{t=1}^T \beta^j u(c_j),$$

where $u(x) = \frac{x^{\rho}}{\rho}$, as desired. First, for every $s^{t-1} = (s_1, \dots, s_{t-1})$, let

$$c_t^1(s^{t-1}) = \left[\mathbb{E}_{P(s_t)}h_t^{\alpha}\right]^{\frac{1}{\alpha}}.$$

Observe that axioms 7 and 8, we have

$$h \sim_0 (h_0, \ldots, h_{T-1}, c_T^1).$$

Now by further applying axioms 7 and 8 we get

$$(h_0,\ldots,h_{T-1},c_T^1)\sim_0 (h_0,\ldots,c_{T-1}^1,c_T^1).$$

By axiom 6,

$$(h_0,\ldots,c_{T-1}^1,c_T^1)\sim_0 (h_0,\ldots,c_{T-1}^1,\hat{c}_T^1),$$

where $\hat{c}_T^1(s_1, s_{T-2}, \ldots, \cdot, s_T)$ is constant and

$$\hat{c}_T^1(s_1,\ldots,\cdot)=c^1(s_1,\ldots,\cdot).$$

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By another application of axioms 7 and 8 we obtain:

$$(h_0,\ldots,c_{T-1}^1,\hat{c}_T^1)\sim_0 (h_0,\ldots,c_{T-1}^1,c_T^2),$$

where

$$c_T^2 = \left[\mathbb{E}_{P(s_{T-1})P(s_T)} h_T^{\alpha} \right]^{\frac{1}{\alpha}}$$

Proceeding as in the previous to steps, we obtain at step t

$$h \sim_0 (h_0, \dots, c_{T-t+1}^1, \dots, c_{T-1}^{t-1}, c_T^t)$$

where

$$c_j^t = \left[\mathbb{E}_{\prod_{\tau=T-t+1}^j P(s_\tau)} h_j^{\alpha} \right]^{\frac{1}{\alpha}}.$$

Specifically, after T steps we get

$$h \sim_0 (h_0, c_1, \ldots, c_T),$$

as desired.

I turn to the necessity of the axioms. It is immediately verified that all axioms are satisfied, except axiom 6. I prove that the representation satisfies indifference to timing. Take h, h' such that for some $\bar{s}^t = (\bar{s}_1, \dots, \bar{s}_t)$ with $1 \le t \le T - 2$ the act

$$(h_0, h_1(\bar{s}_1), h_2(\bar{s}_1, \bar{s}_2), \dots, h_t(\bar{s}^t), \dots, h_T(\bar{s}^t, \cdot)),$$

resolves earlier than

$$(h'_0, h'_1(\bar{s}_1), h'_2(\bar{s}_1, \bar{s}_2), \dots, h'_t(\bar{s}^t), \dots, h'_T(\bar{s}^t, \cdot)),$$

and $h_{\tau}(s_1, \ldots, s_{\tau}) = h'_{\tau}(s_1, \ldots, s_{\tau})$ for every (s_1, \ldots, s_{τ}) with $\tau < t$ and $h_{\tau}(s_1, \ldots, s_{\tau}) = h'_{\tau}(s_1, \ldots, s_{\tau})$ for every (s_1, \ldots, s_{τ}) such that $\tau \ge t$ and $(s_1, \ldots, s_t) \ne (\bar{s}_1, \ldots, \bar{s}_t)$. Then we have for $t' \le t$

$$V_{t'}((s_1,\ldots,s_{t'}),h) - V_{t'}((s_1,\ldots,s_{t'}),h') \propto \sum_{j=0}^{T-t} \beta^j u \bigg(\bigg[\mathbb{E}_{\prod_{\tau=1}^j P(s_{t+\tau})} h_{t+j}^{\alpha} \bigg]^{\frac{1}{\alpha}} \bigg) - \sum_{j=0}^{T-t} \beta^j u \bigg(\bigg[\mathbb{E}_{\prod_{\tau=1}^j P(s_{t+\tau})} h_{t+j}^{\prime \alpha} \bigg]^{\frac{1}{\alpha}} \bigg).$$

Observe that by assumption on h, h' we have

$$\mathbb{E}_{\prod_{\tau=1}^{j} P(s_{t+\tau})} h_{t+j}^{\alpha} = \mathbb{E}_{\prod_{\tau=1}^{j} P(s_{t+\tau})} h_{t+j}^{\alpha},$$

for $j = 0, \ldots, T - t$. Therefore $V_{t'}((s_1, \ldots, s_{t'}), h) - V_{t'}((s_1, \ldots, s_{t'}), h') = 0$ whence the result follows.

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Calculations for Example 1

Given preferences $\succeq_{t,\omega}$ which admit an EZSS representation $(\alpha, \rho, \beta, \mathcal{R})$ with $(\alpha, \rho, \beta) = (0.819, 0.579, 1.193)$, when $\mathcal{R} = \mathcal{F}$ we have that

$$\pi^*(h) = 1 - \frac{\hat{V}_0(\bar{h})}{\hat{V}_0(h)},$$

where

$$\hat{V}_{0}(\bar{h}) = \left(1.193 \left(\frac{1}{2} \cdot \left(\left(100^{0.579} + 1.193 \cdot 170^{0.579}\right)^{\frac{1}{0.579}}\right)^{0.819} + \frac{1}{2} \cdot \left(\left(100^{0.579} + 1.193 \cdot 10^{0.579}\right)^{\frac{1}{0.579}}\right)^{0.819}\right)^{\frac{0.579}{0.819}}\right)^{\frac{1}{0.579}}$$
$$\hat{V}_{0}(h) = \left(1.193 \left(\frac{1}{2} \cdot (U_{2})^{0.819} + \frac{1}{2} \cdot (U_{2})^{0.819}\right)^{\frac{0.579}{0.819}}\right)^{\frac{1}{0.579}}$$

and

$$U_2 = \left(100^{0.579} + 1.193 \cdot \left(\frac{1}{2} \cdot 170^{0.819} + \frac{1}{2} \cdot 10^{0.819}\right)^{\frac{0.579}{0.819}}\right)^{\frac{1}{0.579}}$$

from which we obtain $\pi^*(h) = -0.0418$. In contrast, when $\mathcal{R} = IND$ we have that

$$\hat{V}_{0}(\bar{h}) = \left(1.193 \left(\left(\frac{1}{2} \cdot 100^{0.819} + \frac{1}{2} \cdot 100^{0.819} \right)^{\frac{1}{0.819}} + 1.193^{2} \left(\frac{1}{2} \cdot \frac{1}{2} \cdot 170^{0.819} + \frac{1}{2} \cdot \frac{1}{2} \cdot 10^{0.819} + \frac{1}{2} \cdot \frac{1}{2} \cdot 10^{0.819} + \frac{1}{2} \cdot \frac{1}{2} \cdot 170^{0.819} + \frac{1}{2} \cdot \frac{1}{2} \cdot 10^{0.819} \right)^{\frac{1}{0.579}} \right)^{\frac{1}{0.579}},$$

from which we can find that $\pi^*(h) = 0.347$, so that $\pi^*(h) \approx 4\%$. Finally, if $\mathcal{R} = GR$, then $h, \bar{h} \notin \mathcal{R}$ and so by axiom 6 it follows that $\hat{V}_0(h) = \hat{V}_0(\bar{h})$, implying $\pi^*(h) = 0$.

Calculations for Example 2

In this example, we consider a consumption-savings problem over three periods: t = 0, 1, 2. The agent chooses a consumption plan (c_0, c_1, c_2) and investment

levels (n_f, n_r) in a risk-free asset with return R_f and a risky asset with stochastic return $R_r(s^t)$, where t = 1, 2. The possible states occur with probabilities $P(H) = P(L) = \frac{1}{2}$.

The agent maximizes expected utility:

$$\max_{c \in \mathcal{F}} V_0(c)$$

subject to the following budget constraints:

$$c_{0} = w - (n_{f} + n_{r}) \ge 0,$$

$$c_{1}(s^{1}) = n_{f}R_{f}(s^{1}) + n_{r}R_{r}(s^{1}) - (n_{f}(s^{1}) + n_{r}(s^{1})) \ge 0, \quad s^{1} \in \{H, L\},$$

$$c_{2}(s^{2}) = n_{f}(s^{1})R_{f}(s^{2}) + n_{r}(s^{1})R_{r}(s^{2}) \ge 0, \quad s^{2} \in \{(H, L), (H, H), (L, L), (L, H)\}.$$

By symmetry, one can assume that $n_r(H) = n_r(L)$. Note that if the agent does not invest in the risky asset at the second-period nodes, the consumption plan corresponds to an early resolution program, which falls outside the $\mathcal{R} = GR$ domain. Therefore, under EZSS utility with $\mathcal{R} = GR$, DOCE utility is used to determine optimal demands when the second-period risky investment is zero. In all other cases, EZ utility is used. The final step is to compare the maximum utility under the early resolution program and the maximum utility under the EZ utility program.

In particular, the optimal utility under the assumption that second-period risky investment is zero, i.e., $n_r(H) = n_r(L) = 0$, is 12.75. This value is achieved with the investment levels:

$$n_r(s_0) = 7.94, \quad n_f(s_0) = 0, \quad n_f(H) = n_f(L) = 4.89.$$

In contrast, the unrestricted optimal utility under EZ preferences is 10.51, obtained with:

$$n_r(s_0) = 0$$
, $n_f(s_0) = 4.02$, $n_r(H) = n_r(L) = 0.76$, $n_f(H) = n_f(L) = 1.66$.

Hence, the optimal investment under the assumption $\mathcal{R} = GR$ is:

$$n_r(s_0) = 7.94, \quad n_f(s_0) = 0, \quad n_r(H) = n_r(L) = 0, \quad n_f(H) = n_f(L) = 4.89.$$

Finally, when $\mathcal{R} = \emptyset$, the maximum utility is 12.76, achieved with:

$$n_r(s_0) = 7.95, \quad n_f(s_0) = 0, \quad n_r(H) = n_r(L) = 4.89, \quad n_f(H) = n_f(L) = 0.$$

These investment strategies result in the optimal consumption paths illustrated in Fig.4.

References

- Al-Najjar, N.I., Weinstein, J.: The ambiguity aversion literature: a critical assessment. Econ. Philos. 25(3), 249–284 (2009)
- Amarante, M., Siniscalchi, M.: Recursive maxmin preferences and rectangular priors: a simple proof. Econ. Theory Bull. 7(1), 125–129 (2019)
- Andreasen, M.M., Jørgensen, K.: The importance of timing attitudes in consumption-based asset pricing models. J. Monet. Econ. 111, 95–117 (2020)
- Attanasio, O.P., Weber, G.: Intertemporal substitution, risk aversion and the euler equation for consumption. Econ. J. **99**(395), 59–73 (1989)
- Bommier, A., Kochov, A., Le Grand, F.: On monotone recursive preferences. Econometrica **85**(5), 1433–1466 (2017)
- Caliendo, F.N., Casanova, M., Gorry, A., Slavov, S.: Retirement timing uncertainty: empirical evidence and quantitative evaluation. Rev. Econ. Dyn. 51, 226–266 (2023)
- Chew, S.H., Epstein, L.G. Recursive utility under uncertainty. In: Ali Khan, M., Yannelis, N.C. (eds.) Equilibrium Theory in Infinite Dimensional Spaces, pp. 352–369. Springer (1991)
- Collard, F., Mukerji, S., Sheppard, K., Tallon, J.-M.: Ambiguity and the historical equity premium. Quant. Econ. **9**(2), 945–993 (2018)
- Debreu, G.: Representation of a preference ordering by a numerical function. Decis. Processes **3**, 159–165 (1954)
- Epstein, L.G., Le Breton, M.: Dynamically consistent beliefs must be bayesian. J. Econ. Theory **61**(1), 1–22 (1993)
- Epstein, L.G., Schneider, M.: Recursive multiple-priors. J. Econ. Theory 113(1), 1-31 (2003)
- Epstein, L., Wang, T.: Intertemporal asset pricing under Knightian uncertainty. Econometrica 62(2), 283– 322 (1994)
- Epstein, L.G., Zin, S.E.: Substitution, risk aversion, and the temporal behavior of consumption and asset returns: a theoretical framework. Econometrica **57**(4), 937–969 (1989)
- Epstein, L.G., Zin, S.E.: Substitution, risk aversion, and the temporal behavior of consumption and asset returns: an empirical analysis. J. Polit. Econ. **99**(2), 263–286 (1991)
- Epstein, L.G., Farhi, E., Strzalecki, T.: How much would you pay to resolve long-run risk? Am. Econ. Rev. **104**(9), 2680–97 (2014)
- Ghirardato, P.: Revisiting Savage in a conditional world. Econ. Theory **20**(1), 83–92 (2002). https://doi. org/10.1007/s001990100188
- Gorman, W.M.: The structure of utility functions. Rev. Econ. Stud. 35(4), 367-390 (1968)
- Hall, R.E.: Real interest and consumption. NBER working paper, (w1694) (1985)
- Hammond, P.J.: Consistent plans, consequentialism, and expected utility. Econometrica 57(6), 1445–1449 (1989)
- Hardy, G.H., Littlewood, J.E., Pólya, G.: Inequalities. Cambridge University Press, Cambridge (1952)
- Hayashi, T.: Intertemporal substitution, risk aversion and ambiguity aversion. Econ. Theory **25**(4), 933–956 (2005). https://doi.org/10.1007/s00199-004-0508-2
- Johnsen, T.H., Donaldson, J.B.: The structure of intertemporal preferences under uncertainty and time consistent plans. Econometrica 53(6), 1451–1458 (1985)
- Ju, N., Miao, J.: Ambiguity, learning, and asset returns. Econometrica 80(2), 559–591 (2012)
- Karantounias, A.G.: Optimal fiscal policy with recursive preferences. Rev. Econ. Stud. 85(4), 2283–2317 (2018)
- Kreps, D.M., Porteus, E.L.: Temporal resolution of uncertainty and dynamic choice theory. Econometrica 46(1), 185–200 (1978)
- Kubler, F., Selden, L., Wei, X.: Time consistency, temporal resolution indifference and the separation of time and risk. Columbia Business School Research Paper (2019)
- Lu, J., Luo, Y., Saito, K., Xin, Y.: Did Harold Zuercher have time-separable preferences? (2024). arXiv preprint arXiv:2406.07809
- Marinacci, M., Principi, G., Stanca, L.: Recursive preferences and ambiguity attitudes (2023). arXiv preprint arXiv:2304.06830
- Marinacci, M., Montrucchio, L.: Unique solutions for stochastic recursive utilities. J. Econ. Theory 145(5), 1776–1804 (2010)
- Martin, I.W.: Consumption-based asset pricing with higher cumulants. Rev. Econ. Stud. **80**(2), 745–773 (2013)

- McClennen, E.F.: Rationality and Dynamic Choice: Foundational Explorations. Cambridge University Press, Cambridge (1990)
- Meissner, T., Pfeiffer, P.: Measuring preferences over the temporal resolution of consumption uncertainty. J. Econ. Theory 200, 105379 (2022)
- Sarin, R., Wakker, P.P.: Dynamic choice and nonexpected utility. J. Risk Uncertain. 17, 87-120 (1998)

Savage, L.: Foundations of Statistics. Wiley, New York (1954)

- Selden, L.: A new representation of preferences over "certain x uncertain" consumption pairs: the "ordinal certainty equivalent" hypothesis. Econom. J. Econom. Soc. 1045–1060 (1978)
- Selden, L., Stux, I.E.: Consumption Trees, OCE Utility and the Consumption/Savings Decision. Columbia University, Graduate School of Business (1978)
- Skiadas, C.: Recursive utility and preferences for information. Econ. Theory 12(2), 293–312 (1998). https:// doi.org/10.1007/s001990050222
- Stokey, N.L., Lucas, R.E.: Recursive Methods in Economic Dynamics. Harvard University Press, Cambridge (1989)
- Strzalecki, T.: Temporal resolution of uncertainty and recursive models of ambiguity aversion. Econometrica 81(3), 1039–1074 (2013)
- Suzuki, S., Yamagami, H.: On the effects of pessimism toward pollution-driven disasters on equity premiums. Econ. Theory Bull. 12(2), 167–181 (2024)
- Wakai, K.: Recursive extension of a multicommodity analysis. Econ. Theory Bull. 3(2), 271-285 (2015)
- Wang, T.: Conditional preferences and updating. J. Econ. Theory 108(2), 286-321 (2003)
- Weller, P.: Consistent intertemporal decision-making under uncertainty. Rev. Econ. Stud. **45**(2), 263–266 (1978)
- Zin, S.E. et al.: Intertemporal substitution, risk and the time series behaviour of consumption and asset returns. Technical report (1987)

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