

# Recursive Preferences, Correlation Aversion, and the Temporal Resolution of Uncertainty\*

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## Abstract

This paper investigates a behavioral feature of recursive preferences: aversion to risks that are persistent through time. I introduce a formal notion of correlation aversion to capture this phenomenon and show that increasing relative risk aversion (IRRA) is equivalent to correlation aversion. I further show that IRRA constrains preferences for early resolution of uncertainty. However, I demonstrate that common models of recursive preferences such as Epstein-Zin cannot distinguish risk aversion from correlation aversion and preferences for early resolution of uncertainty, leading to empirical issues in asset pricing. I propose a generalization of the Epstein-Zin model that can accommodate such difficulties. Finally, I show that correlation averse preferences admit a variational representation, which connects correlation aversion to fear of model misspecification.

Keywords: Intertemporal substitution, risk aversion, correlation aversion, recursive utility, preference for early resolution of uncertainty, information.

JEL classification: C61, D81.

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# 1 Introduction

Recursive preferences are of central importance in many economic applications. They play a key role in models of consumption-based asset pricing (Epstein and Zin 1989, Epstein and Zin 1991), precautionary savings (Weil 1989, Hansen et al. 1999), business cycles (Tallarini 2000), and risk-sharing (Epstein 2001, Anderson 2005); more recently they have been applied to climate change (Bansal et al. 2017, Cai and Lontzek 2019), optimal fiscal policy (Karantounias 2018), and repeated games (Kochov and Song 2021).

Recursive preferences can distinguish between risk aversion and intertemporal substitution. Indeed, recursive preferences admit a representation in terms of utilities  $(V_t)_t$  which satisfy the recursive relation

$$V_t = u(c_t) + \beta\phi^{-1}(\mathbb{E}_p\phi(V_{t+1})), \quad (1)$$

where  $u$  reflects intertemporal substitution,  $\phi$  is an adjustment factor which reflects attitudes toward risk,  $\beta$  is the discount factor, and  $p$  describes the distribution of future consumption.<sup>1</sup> In the standard model,  $\phi$  is the identity function so that risk aversion and attitudes toward consumption smoothing are both captured by the curvature of  $u$  and therefore they cannot be identified separately from each other. From Kreps and Porteus (1978), we know that disentangling these aspects implies a preference for early resolution of uncertainty or, equivalently, a preference for non-instrumental information.

The present paper introduces a new axiom called *correlation aversion*, as it requires aversion to persistent consumption shocks. In the dynamic setting I consider, this axiom imposes restrictions on an individual's willingness to pay for *non-instrumental information* about future consumption. I provide bounds—based on risk attitudes—to the demand for non-instrumental information that are necessary and sufficient for recursive preferences to satisfy correlation aversion. However, recursive models such as Epstein-Zin preferences face challenges in distinguishing risk aversion from correlation aversion and preferences for non-instrumental information, leading to empirical complications in asset pricing. I propose a generalization of the Epstein-Zin model that can accommodate such difficulties.

To illustrate, consider two gambles:  $A$  and  $B$ . In gamble  $A$ , a fair coin is tossed at

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<sup>1</sup>See Cochrane (2009), Chapter 21.3 or Campbell (2017), Chapter 6.4 for a textbook treatment.

$t = 1$ . If the outcome is heads, then consumption is constant at the level 1 for every following period. Otherwise, it is constant at the level 0 at every period. In gamble  $B$ , consumption is determined by tossing a fair coin at every period, giving a level of consumption equal to 1 if heads and 0 otherwise. A hedging motive suggests that a decision maker should prefer  $B$  to  $A$ . But at the same time  $B$  resolves gradually while for  $A$  all risk resolves at  $t = 1$ . As a result, gamble  $A$  provides more non-instrumental information than  $B$  regarding future consumption.<sup>2</sup> The comparison between these two gambles is therefore non-obvious:  $A$  has the advantage of resolving all risk at  $t = 1$ , while  $B$  is more desirable because of its hedging value.

My notion of correlation aversion requires the hedging motive to dominate the preference for non-instrumental information so that a decision maker prefers  $B$  to  $A$ . I consider a setting in which preferences are defined over temporal lotteries. I introduce a novel notion for comparing the correlation of temporal lotteries. A temporal lottery is more correlated than another if it displays a higher degree of (positive) correlation between consumption at two different time periods. Proposition 4 shows that increasing correlation correspondingly increases (non-instrumental) informativeness in the Blackwell sense. This result formalizes the trade-off between intertemporal hedging and non-instrumental information.

The main result, Theorem 1, shows a decision maker is always averse to increasing correlation if and only if  $\phi$  satisfies increasing relative risk aversion (IRRA). Notably, IRRA includes the Epstein-Zin and Hansen-Sargent formulations of recursive preferences as special cases, which are common in applications. I further show that IRRA puts bounds to the demand for non-instrumental information. In other words, it sets a limit on how much a decision maker with these risk attitudes values non-instrumental information. Theorem 1 shows that this bound is strong enough that the value of intertemporal hedging always dominates the value of non-instrumental information.

**Asset pricing.** This result has important implications for the literature on consumption-based asset pricing. The long-run risk model of [Bansal and Yaron \(2004\)](#) explains several puzzles in the asset pricing literature, including the equity premium puzzle. In this model, consumption growth contains a small, persistent component. This persistence amplifies the equity premium such that it matches the observed values in

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<sup>2</sup>Here we are referring to risk about consumption and not about income. Therefore, there is no planning advantage to tossing the coin early: information is non-instrumental.

the data.

Persistence implies a trade-off between intertemporal hedging and non-instrumental information about consumption growth for an investor with Epstein-Zin preferences. By Theorem 1, aversion to this persistent component entails a restriction on preferences for non-instrumental information. Therefore, the equity premium in this model is higher relative to the discounted expected utility benchmark because Epstein-Zin preferences satisfy IRRA and therefore correlation aversion, *despite* exhibiting a preference for non-instrumental information.<sup>3</sup>

This analysis indicates the need for calibrating preference parameters to achieve a reasonable level of correlation aversion. To do that, I follow the approach of Epstein et al. (2014) and introduce the notion of *persistence premium*, which is based on the following question: “What fraction of your wealth would you give up to remove all persistence in consumption growth?” I show that, under standard parameter specifications, an investor’s persistence premium is not consistent with the experimental evidence.

I argue that this issue and the puzzle regarding the timing premium in Epstein et al. (2014) arise due to the Epstein-Zin model’s limitation in fully separating three distinct features of preferences: correlation aversion, risk aversion, and preferences for early resolution of uncertainty. Since correlation aversion is determined entirely by the level of risk aversion, an excessive amount of risk aversion is required to match the observed equity premium.

To address this problem, I discuss a generalization of Epstein-Zin preferences that achieve a partial disentangling of these three features of preferences. These preferences satisfy the recursion

$$V_t = u(c_t) + \beta(c)\phi^{-1}(\mathbb{E}_p\phi(V_{t+1})), \quad (2)$$

where  $\phi$  exhibits hyperbolic absolute risk aversion (HARA) and  $\beta(c)$  is the endogenous discount factor that depends on the level of consumption.

**Connection with model misspecification.** To elucidate further the role played by risk attitudes in Theorem 1, I show in Theorem 2 that under a mild strengthening

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<sup>3</sup>Compare this point with the common understanding of the long-run risk model. For example, Bansal et al. (2016) state “The long-run risks (LRR) asset pricing model emphasizes the role of low-frequency movements [...] along with investor preferences for early resolution of uncertainty, as an important economic-channel that determines asset prices”.

of IRRA, recursive preferences admit the variational representation

$$V_t = u(c_t) + \beta \left[ \min_q \mathbb{E}_q V_{t+1} + I_{\phi, u, \beta}^t(q||p) \right], \quad (3)$$

where  $I_{\phi, u, \beta}^t(q||p)$  is a (generalized) statistical distance that measures the dissimilarity of any other distribution of future consumption  $q$  from  $p$ . The interpretation is that the decision maker does not fully trust the distribution of future consumption given by the reference probability  $p$ . Instead, all other possible distributions  $q$  are considered plausible and evaluated depending on their dissimilarity from  $p$  as measured by  $I_{\phi, u, \beta}^t(q||p)$ .

A strand of the literature (e.g., [Hansen et al. 1999](#)) has motivated the use of models of recursive utility with robustness concerns and in particular fear of model misspecification. The representation in (3) provides a general connection between correlation aversion and robustness to model misspecification.<sup>4</sup>

When  $\phi$  satisfies constant absolute risk aversion,  $I_{\phi, u, \beta}^t(q||p)$  is given by relative entropy (as shown by [Strzalecki 2011](#)). [Theorem 2](#) generalizes this connection with fear of model misspecification for a much broader class of preferences that satisfy correlation aversion. In the Epstein-Zin case,  $I_{\phi, u, \beta}^t(q||p)$  is defined in terms of the Rényi divergence, a common type of measure of divergence between probability measures that has application in several fields.

**Related literature.** The theoretical literature on dynamic choice has considered a notion of correlation aversion derived from the literature on risk aversion with multiple commodities started by [Kihlstrom and Mirman \(1974\)](#) (see also [Richard 1975](#) or [Epstein and Tanny 1980](#)). In particular, [Bommier \(2007\)](#) considers a notion of correlation aversion based on the [Kihlstrom and Mirman](#) approach in a continuous time setting. [Kochov \(2015\)](#) and [Bommier et al. \(2019\)](#) study the extension to a purely subjective setting of this property, which they refer to as intertemporal hedging. Intertemporal hedging involves comparing intertemporal gambles that do not differ in terms of temporal resolution of uncertainty (see [Section 3](#) for a discussion). [Miao and Zhong \(2015\)](#) and [Andersen et al. \(2018\)](#) relate Epstein-Zin utility to an analogous notion of intertemporal hedging and provide experimental evidence in its favor. In

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<sup>4</sup>To better appreciate the connection with fear of model misspecification, observe that gamble  $A$  is like flipping a biased coin with uncertainty about the bias, while for gamble  $B$  there is no such uncertainty. Preferring  $B$  to  $A$  reveals aversion to model misspecification.

the Supplemental Appendix (see Section S.1), I show that within the class of recursive preferences in (1)—which I refer to as Kreps-Porteus preferences—intertemporal hedging is equivalent to  $\phi$  being concave, i.e. risk aversion.

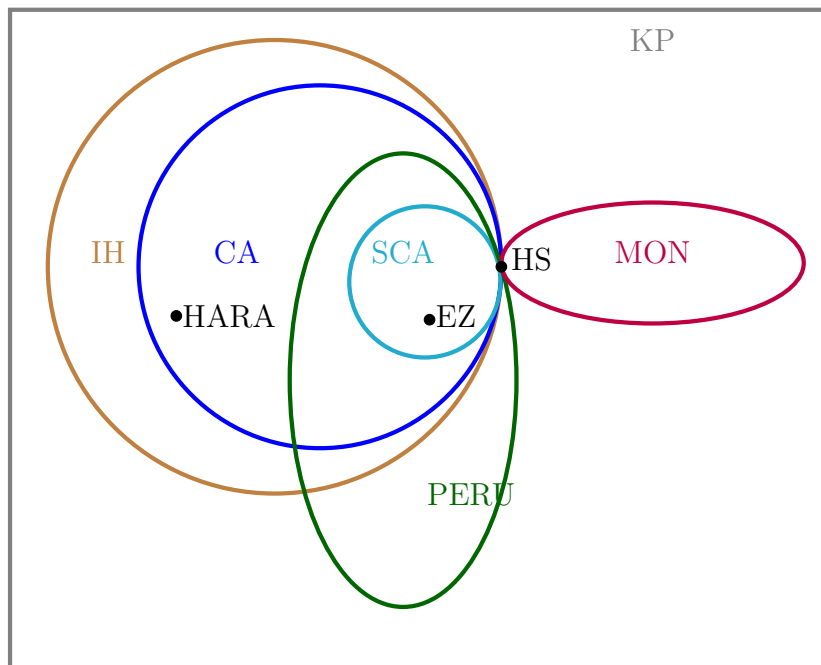


Figure 1: Relationship between correlation averse (CA) preferences and other recursive risk preferences: recursive preferences that satisfy intertemporal-hedging (IH), Epstein-Zin (EZ) preferences, HARA preferences, multiplier-preferences (HS), monotone recursive preferences (MON), preferences that exhibit a preference for early resolution of uncertainty (PERU), and strong correlation aversion (SCA). HS preferences are the only ones that exhibit all these features at the same time.

I also consider the notion of strong correlation aversion, which strengthens IRRRA and gives the representation in (3). Epstein and Zin’s (1989) preferences and Hansen and Sargent’s (2001) multiplier preferences satisfy this condition. Within the Kreps-Porteus setting, multiplier preferences are the only ones to jointly satisfy strong correlation aversion, preferences for early resolution of uncertainty, and monotonicity as defined in Bommier et al. (2017). In contrast, HARA preferences as in (2) do not satisfy strong correlation aversion. However, they can (partially) disentangle risk aversion from correlation aversion and preferences for non-instrumental information.

Figure 1 illustrates the relationship just discussed between correlation aversion

and other prominent classes of recursive preferences. I discuss the relationship of correlation aversion with the work of [DeJarnette et al. \(2020\)](#) and [Dillenberger et al. \(2020\)](#) on preferences that satisfy stochastic impatience in more depth in Section 4. Section 4 also discusses the relationship between correlation aversion and applications of recursive preferences to climate policy and optimal fiscal policy.

**Organization of the paper.** Section 2 introduces the notation and the main choice-theoretic objects used in the paper, and provides a novel treatment of preference for early resolution of uncertainty. This treatment is used in Section 3 to establish the main results related to correlation aversion. Section 3.1 discusses the implications for asset pricing, and Section 3.2 provides the connection between correlation aversion and model misspecification. Section 4 contains an additional discussion of the related literature. Section 5 concludes the paper. The proofs are in the Appendix, which is completed by the Supplemental Appendix.

## 2 Preliminaries

**Choice setting.** I assume that time is discrete and varies over a finite horizon  $2 \leq T < \infty$ . The Supplemental Appendix (see Section S.3) describes the setting with an infinite horizon, i.e.  $T = \infty$ . The consumption set  $C$  is assumed to be  $C = [0, \infty)$ .<sup>5</sup> Given a Polish space  $X$ , let  $\Delta_s(X), \Delta_b(X)$  denote the space of simple and Borel probability measures with bounded support over  $X$ , respectively. Observe that  $\Delta_s(X) \subseteq \Delta_b(X)$  and both are mixture spaces.

Given  $\ell, m \in \Delta_b(X)$  such that  $\ell$  is absolutely continuous with respect to  $m$ , indicated by  $\ell \ll m$ ,  $\frac{d\ell}{dm}$  denotes the Radon-Nikodym derivative. Endow  $\Delta_b(X)$  with the weak\* topology. Given  $x \in X$ , I denote with  $\delta_x \in \Delta_b(X)$  the Dirac probability defined by  $\delta_x(A) = 1$  when  $x \in A$  and  $\delta_x(A) = 0$  when  $x \notin A$ . I denote with  $\bigoplus_{i=1}^n \pi_i m_i$  the mixture of  $n$  probabilities  $(m_i)_{i=1}^n$  in  $\Delta_b(X)$  with a probability vector  $(\pi_i)_{1 \leq i \leq n}$ . Further, note that every two-stage lottery  $m \in \Delta_s(\Delta_s(X))$  can be (uniquely up to permutations) associated to a stochastic matrix  $M[m]$  whose rows describe each probability  $M[m](\cdot|i) \in \text{supp} m$  in the support of  $m$  for  $i = 1, \dots, |\text{supp} m|$ , and to a probability vector  $\mu[m]$  which describes the probability of each  $i$ .

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<sup>5</sup>The same results apply if the consumption set is an open interval, say  $C = (0, \infty)$ .

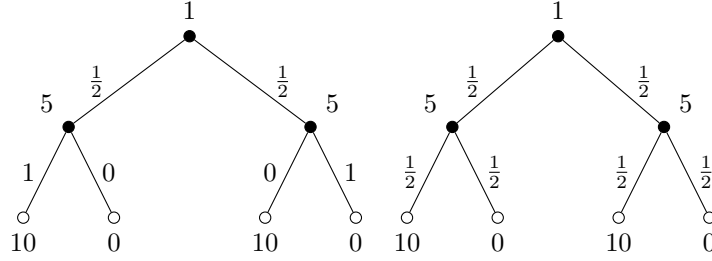


Figure 2: Probability tree representation of two temporal lotteries with  $T = 2$

Temporal lotteries  $(D_t)_{t=0}^T$  are defined by  $D_T := C$  and recursively,

$$D_t := C \times \Delta_b(D_{t+1}),$$

for every  $t = 0, \dots, T - 1$ . Likewise, simple temporal lotteries are defined by  $D_{T,s} := C$ ,  $D_{t,s} := C \times \Delta_s(D_{t+1,s})$  for every  $t = 0, \dots, T - 1$ . Simple temporal lotteries can be intuitively represented using a tree diagram, as illustrated in Figure 2.

I write  $(c_0, (c_1, m)) \in D_0$  for a temporal lottery that consists of two periods of deterministic consumption,  $c_0$  and  $c_1$ , followed by the lottery  $m \in \Delta_b(D_2)$ . More generally, for any consumption vector  $c^t = (c_0, \dots, c_{t-1}) \in C^t$  and  $m \in \Delta_b(D_t)$ , the temporal lottery  $(c_0, (c_1, (c_2, (\dots, (c_{t-1}, m)))))) \in D_0$  or  $(c^t, m)$  for brevity is one that consists of  $t$  periods of deterministic consumption followed by the lottery  $m$ . Given two Polish spaces  $X, Y$  and  $m \in \Delta_b(X \times Y)$  I denote with  $\text{marg}_X m$  the marginal probability over  $X$ , i.e.,  $\text{marg}_X m(A) = m(A \times Y)$  for every measurable set  $A \subseteq X$ .

**Example 1.** Assume  $T = 2$ . Let  $d = (c_0, m) = \left(1, \frac{1}{2}(5, 10) \oplus \frac{1}{2}(5, 0)\right)$  and  $d' = (c_0, m') = \left(1, 5, \left(\frac{1}{2}10 \oplus \frac{1}{2}0\right)\right)$ . Figure 2 provides a graphical representation of these two temporal lotteries. We have  $|\text{supp}m| = |\text{supp}m'| = 2$  and

$$M \left[ \text{marg}_{\Delta_s(D_{2,s})} m' \right] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \text{ and } M \left[ \text{marg}_{\Delta_s(D_{2,s})} m \right] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Moreover,  $\mu[m] = \mu[m'] = \left[\frac{1}{2} \quad \frac{1}{2}\right]$ . △

The preferences of a decision maker over temporal lotteries are given by a collection  $(\succeq_t)_{t=0}^T$  where each  $\succeq_t$  is a weak order over  $D_t$  and  $\succ_t$  denotes the asymmetric part of  $\succeq_t$ . To ease notation, I denote with  $\succeq := (\succeq_t)_{t=0}^T$  the entire collection of preferences. I consider preferences that admit the following general recursive representation described in (1).



**Definition 1.** Preferences  $\succeq$  admit a Kreps-Porteus (KP) recursive representation  $(\phi, u, \beta)$  if each  $\succeq_t$  is represented by  $V_t : D_t \rightarrow \mathbb{R}$  such that  $V_T(c) = u(c)$  for every  $c \in C$  and recursively

$$V_t(c, m) = u(c) + \beta\phi^{-1}(\mathbb{E}_m\phi(V_{t+1})) \quad \text{for } t = 0, \dots, T-1,$$

where  $\beta \in (0, 1]$  is the discount factor,  $u : C \rightarrow \mathbb{R}$  is unbounded above, continuous, and strictly increasing, and  $\phi : u(C) \rightarrow \mathbb{R}$  is a continuous and strictly increasing function.

This representation of preferences effectively separates risk aversion (as captured by the function  $\phi$ ) from intertemporal substitution (as modeled by the utility function  $u$ ). The axiomatic foundation of this representation is well known (see for example Proposition 4 in [Sarver 2018](#)). The parameter  $\beta$  is unique, while  $u$  is cardinally unique and  $\phi$  is cardinally unique given  $u$ .<sup>6</sup> Because  $u$  is unbounded above, one can set without loss of generality  $u(C) = [0, \infty)$ .

This class of preferences comprises many common cases used in applications. Two notable examples are Epstein-Zin preferences (EZ), given by  $u(x) = \frac{x^\rho}{\rho}$  for every  $x \in u(C)$  and  $\phi(x) = \frac{\rho}{\alpha}x^{\frac{\alpha}{\rho}}$  for every  $x \in u(C)$ , where  $0 \neq \alpha < 1, 0 \neq \rho < 1$  and  $\alpha < \rho$ ; Hansen-Sargent multiplier preferences (HS) are given by  $\phi(x) = -\exp\left(-\frac{x}{\theta}\right)$  with  $0 < \theta < \infty$  for every  $x \in u(C)$ .<sup>7</sup>

I will typically consider KP representations with  $\phi$  that is concave and satisfies certain differentiability assumptions to employ standard tools from the theory of risk aversion. Write  $\phi \in \mathcal{C}^r$  if  $\phi$  is continuous and has  $r$  continuous derivatives. Given  $\phi \in \mathcal{C}^2$ , the Arrow-Pratt index  $A_\phi : \text{int } u(C) \rightarrow \mathbb{R}$  is given by

$$A_\phi(x) = -\frac{\phi''(x)}{\phi'(x)} \quad \text{for every } x \in \text{int } u(C),$$

and the index of relative risk aversion defined by  $R_\phi(x) = xA_\phi(x)$  for every  $x \in \text{int } u(C)$ . A function  $\phi$  is decreasing absolute risk averse (DARA) if  $A_\phi$  is non-increasing, it is increasing absolute risk averse (IARA) if its index  $A_\phi$  is nondecreasing, and it is constant absolute risk averse (CARA) if it is both DARA and IARA.

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<sup>6</sup>In the applied literature, the term  $\phi$  is often referred to as *risk adjustment*. See for example [Hansen et al. \(2007\)](#).

<sup>7</sup>Under the present taxonomy, EZ preferences do not overlap with HS preferences, but they would if one allowed for  $\rho = 0$ ; see for example [Hansen et al. \(2007\)](#), Example 2.3.

Decreasing (DRRA), increasing (IRRA), and constant (CRRA) relative risk averse functions are defined analogously by replacing the index  $A_\phi$  with  $R_\phi$ .

## 2.1 Preferences for (non-instrumental) information

I reframe the theory of preferences for early resolution of uncertainty using the language of information economics. Temporal lotteries are partially ordered by means of a version of Blackwell order, which allows comparing them in terms of their (non-instrumental) informativeness. In addition to its theoretical appeal and generality, this approach permits building a formal link between correlation and information.

**Definition 2.** *Given  $d, d' \in D_{0,s}$  say that  $d$  is more informative than  $d'$ , denoted  $d \geq_B d'$ , if there exists  $t \leq T - 2$ ,  $c^t \in C^t$  and  $m, m' \in \Delta_s(C \times \Delta_s(D_{t+1,s}))$  such that  $d = (c^t, m)$ ,  $d' = (c^t, m')$ ,  $\text{marg}_C m' = \text{marg}_C m$  and*

$$M \left[ \text{marg}_{\Delta_s(D_{t+1,s})} m' \right] = GM \left[ \text{marg}_{\Delta_s(D_{t+1,s})} m \right], \quad (4)$$

where  $G$  is a stochastic matrix (i.e., each row of  $G$  forms a probability vector).

In words, the expression  $d \geq_B d'$  means that the two lotteries,  $d$  and  $d'$ , have the same distribution of consumption in period  $t + 1$ . However, the actual realization of consumption in period  $t + 1$  provides more information about future values of consumption (from period  $t + 2$  onwards) for the lottery  $d$  compared to the lottery  $d'$ . Observe that  $\geq_B$  is a partial order just like the standard Blackwell order. The following examples help to further clarify this notion of comparative information.

**Example 2.** Assume  $T = 2$ . Let  $d = \left(1, \frac{1}{2}(5, 10) \oplus \frac{1}{2}(5, 0)\right)$  and  $d' = \left(1, 5, \left(\frac{1}{2}10 \oplus \frac{1}{2}0\right)\right)$ . Figure 2 provides a graphical representation of these two temporal lotteries. We have

$$M \left[ \text{marg}_{\Delta_s(D_{2,s})} m' \right] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} M \left[ \text{marg}_{\Delta_s(D_{2,s})} m \right],$$

so that  $d \geq_B d'$ . In words, the terminal value of consumption is fully revealed by a coin toss at  $t = 1$  for  $d$  but revealed at  $t = 2$  only for  $d'$ . △

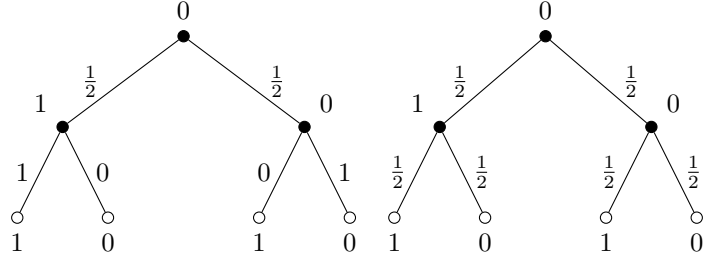


Figure 3: Probability tree representation of a temporal lottery

**Example 3.** Again assume  $T = 2$ . Consider  $d, d'$  given by  $d = \left(1, \frac{1}{2}(1, 1) \oplus \frac{1}{2}(0, 0)\right)$  and  $d' = \left(1, \frac{1}{2}\left(1, \left(\frac{1}{2}1 \oplus \frac{1}{2}0\right)\right) \oplus \frac{1}{2}\left(0, \left(\frac{1}{2}1 \oplus \frac{1}{2}0\right)\right)\right)$ . Figure 3 provides a graphical representation of these two temporal lotteries. We have

$$M \left[ \text{marg}_{\Delta_s(D_{2,s})} m' \right] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} M \left[ \text{marg}_{\Delta_s(D_{2,s})} m \right],$$

so that  $d \geq_B d'$ . In words,  $d'$  is an “iid” temporal lottery while  $d$  is perfectly correlated.  $\triangle$

This notion of comparative information is extended to arbitrary temporal lotteries by means of the following standard procedure.

**Definition 3.** For every  $d, d' \in D_0$ , write  $d \geq_B d'$  if there exist sequences  $(d_n)_{n=0}^\infty, (d'_n)_{n=0}^\infty$  in  $D_{0,s}$  such that  $\lim_n d_n = d$ ,  $\lim_n d'_n = d'$  and  $d_n \geq_B d'_n$  for every  $n \geq 0$ .

A preference for non-instrumental information (or for early resolution of uncertainty) over a domain of temporal lotteries is defined as monotonicity with respect to the order  $\geq_B$ .

**Axiom 1.**  $\succeq$  exhibits a preference for information over a set  $S \subseteq D_0$  if for every  $d, d', \in S$

$$d \geq_B d' \implies d \succeq_0 d'.$$

The [Kreps and Porteus](#)'s approach restricts attention to temporal lotteries such as those in [Example 2](#) in which the draw at  $t = 1$  is deterministic.<sup>8</sup> In this way

<sup>8</sup>For a more recent treatment see for example Definition 2 in [Bommier et al. \(2017\)](#).

consumption at  $t = 1$  is informative but not correlated with consumption at  $t = 2$ . Formally, in this case the set  $S$  is given by

$$S := \left\{ (c^t, m) \in D_{0,s} : \text{there exists } \bar{c} \in C \text{ such that } \text{marg}_C m = \delta(\bar{c}) \right\}. \quad (5)$$

Observe that for these temporal lotteries, consumption at time  $t+1$  is deterministic at level  $\bar{c}$ , which implies that it is uncorrelated with consumption in the following periods. The next result characterizes preferences valuing non-instrumental information over  $B$ .

**Proposition 1.** *Assume  $\succeq$  admits a KP representation  $(\phi, u, \beta)$  with  $\phi \in \mathcal{C}^2$ . Then  $\succeq$  exhibits a preference for information over  $S$  if and only if*

$$-\beta \frac{\phi''(\beta x + y)}{\phi'(\beta x + y)} \leq -\frac{\phi''(x)}{\phi'(x)}, \quad (6)$$

for every  $x, y \in \text{int } u(C)$ .<sup>9</sup>

*Proof.* See the Appendix. □

The quantity

$$-\frac{\phi''(x)}{\phi'(x)} + \beta \frac{\phi''(\beta x + y)}{\phi'(\beta x + y)},$$

can be considered as a local measure of strength of preferences for non-instrumental information. For example, when  $\beta = 1$  and  $\phi(x) = -\exp\left(-\frac{x}{\theta}\right)$  we obtain

$$-\frac{\phi''(x)}{\phi'(x)} + \beta \frac{\phi''(\beta x + y)}{\phi'(\beta x + y)} = \frac{1}{\theta} - \frac{1}{\theta} = 0,$$

which implies indifference to non-instrumental information. The same applies if  $\phi$  is the identity.

Here I focus on risk attitudes that exhibit a preference for information regardless of the level of impatience or intertemporal substitution.

**Definition 4.** *Say that  $\phi$  satisfies a uniform preference for information (UPI) if every  $\succeq$  with KP representation  $(\phi, u, \beta)$  exhibits a preference for information.*

The next simple result provides a connection between classical risk attitudes and preference for information.

**Proposition 2.** *If  $\phi \in \mathcal{C}^2$  satisfies UPI then it also satisfies DARA.*

*Proof.* Immediate from (6). □

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<sup>9</sup>Condition (6) is due to [Strzalecki \(2013\)](#); (see p. 1051).

### 3 Main results: correlation aversion

I introduce a general notion of an increase in positive correlation between consumption at two distinct periods. I then characterize recursive preferences that are averse to correlation. For ease of exposition, I consider the case in which there are two risky periods, i.e.  $T = 2$ . The Supplemental Appendix (see Sections S.2 and S.3) extends the results to the arbitrary finite horizon ( $T < \infty$ ) and infinite horizon  $T = \infty$ , showing in particular how the case  $T = 2$  can be extended to introduce persistence over time to study long-run risk, i.e. persistence over multiple periods.

I introduce a class of temporal lotteries that can be defined by (i) the distribution of consumption at time  $t = 1$  and (ii) the conditional distribution of consumption at time  $t = 2$  given consumption in the previous period. The advantage is that lotteries within this class can be ordered based on their correlation.

Let

$$M_s^* := \{m \in \Delta_s(C \times \Delta_s(C)) : (c, \mu), (c, \mu') \in \text{supp } m \implies \mu = \mu'\}.$$

Every such  $m \in M_s^*$  can be (uniquely) associated with  $m_1 \in \Delta_s(C)$  and  $m_2(\cdot|\cdot) \in \Delta_s(C)^{\text{supp } m_1}$ , defined by  $m_1 = \text{marg}_C m$ , and

$$m_2(\cdot|c) = \mu(\cdot),$$

where  $\mu$  is the unique element of  $\Delta_s(C)$  such that  $(c, \mu) \in \text{supp } m$ . Conversely, given  $m_1 \in \Delta_s(C)$  and  $m_2(\cdot|\cdot) \in \Delta_s(C)^{\text{supp } m_1}$ , we can uniquely define  $m \in M_s^*$  by

$$m(c, m_2(\cdot|c)) := m_1(c) \quad \text{for every } c \in \text{supp } m_1.$$

In words,  $m_1$  describes the distribution of time 1 consumption while  $m_2(\cdot|c)$  is the conditional distribution of consumption at the final time period given a realization of  $t = 1$  consumption. The set  $D_{0,s}^* := \{(c, m) \in D_{0,s} : m \in M_s^*\}$  is the set of temporal lotteries that can be described in terms of a pair  $(m_1, m_2)$ . Likewise, one can define the associated cumulative distributions  $m_1(c_1 \leq \cdot)$ ,  $m_2(c_2 \leq \cdot|c_1 \leq \cdot)$ .

The structure of these lotteries can be used to introduce the following notion of increasing correlation.

**Definition 5.** Consider  $d = (c_0, m), d' = (c_0, m') \in D_{0,s}^*$ . Say that  $d$  differs from  $d'$  by an *intertemporal elementary correlation increasing transformation*

(IECIT) if and only if  $m_1 = m'_1$  and there exist  $\varepsilon \geq 0$  and a pair  $(c, c')$  such that  $c \neq c', m_1(c), m_1(c') \neq 0$  and

$$\begin{aligned} m_2(c|c) &= m'_2(c|c) + \frac{\varepsilon}{m'_1(c)}, \\ m_2(c'|c) &= m'_2(c'|c) - \frac{\varepsilon}{m'_1(c)}, \\ m_2(c|c') &= m'_2(c|c') - \frac{\varepsilon}{m'_1(c')}, \\ m_2(c'|c') &= m'_2(c'|c') + \frac{\varepsilon}{m'_1(c')}, \end{aligned}$$

and  $m_2 = m'_2$  otherwise.

In simpler terms, these transformations increase the probability that, if consumption at  $t = 1$  is either  $c$  or  $c'$ , it will remain the same at  $t = 2$  and concurrently decrease the probability that consumption will shift to a different level. The following two examples serve as an illustration of this concept.

**Example 4** (Example 3 continued). In this case we have  $m_1 = m'_1$ ,  $m_2(1|1) = 1 = m'_2(1|1) + \frac{1}{1/2} \frac{1}{4} = \frac{1}{2} + \frac{1}{2}$ ,  $m_2(1|1) = 0 = m'_2(1|1) - \frac{1}{1/2} \frac{1}{4} = \frac{1}{2} - \frac{1}{2}$ ,  $m_2(1|0) = 0 = m'_2(1|0) - \frac{1}{1/2} \frac{1}{4} = \frac{1}{2} - \frac{1}{2}$  and  $m_2(0|0) = 1 = m'_2(0|0) + \frac{1}{1/2} \frac{1}{4} = \frac{1}{2} + \frac{1}{2}$ . It follows that  $d$  differs from  $d'$  by an IECIT with  $\varepsilon = \frac{1}{4}$ . Therefore, the perfectly correlated temporal lottery  $d$  can be obtained from the “iid” lottery  $d'$  by means of an IECIT. In this case, an IECIT also increases the informativeness of a temporal lottery.  $\triangle$

The concept of an IECIT is an application of Epstein and Tanny’s (1980) idea of generalized increasing correlation, applied in a dynamic setting. With the notion of an IECIT, it is possible to establish an ordering  $\geq_C$  that can be used to rank temporal lotteries based on their positive autocorrelation.

**Definition 6.** Given  $d, d' \in D_{0,s}^*$  say that  $d$  is more correlated than  $d'$ , denoted  $d \geq_C d'$ , if  $d$  differs from  $d'$  by a finite amount of IECITs.

I provide a necessary condition of when two temporal lotteries differ by a finite amount of IECITs.

**Proposition 3.** If  $d \geq_C d'$  then it holds that

$$m'_2(c_2 \leq c \mid c_1 \leq c') \leq m_2(c_2 \leq c \mid c_1 \leq c') \text{ for every } (c, c') \in C \times C.$$

Notably, Proposition 3 implies that  $\geq_C$  is transitive and thus a partial order. Finally, denote with  $D_0^*$  the weak\* closure of  $D_{0,s}^*$ . It is possible to extend  $\geq_C$  to  $D_0^*$  as follows.

**Definition 7.** Given  $d = (c, m), d' = (c, m') \in D_0^*$ , say that  $d$  is more correlated than  $d'$ , written  $d \geq_C d'$ , if there exist sequences  $(d_n)_{n=0}^\infty, (d'_n)_{n=0}^\infty$  in  $D_{0,s}^*$  such that  $\lim_n d_n = d, \lim_n d'_n = d'$  and  $d_n \geq_C d'_n$  for every  $n \geq 0$ .

The following result establishes a formal connection between IECITs and non-instrumental information by showing that increasing the correlation of “iid” temporal lotteries makes them more informative. To this end, define the “iid” temporal lottery for each  $\ell \in \Delta_b(C)$  by  $d^{iid}(\ell) = (c, m)$  where  $m(A \times B) = \ell(A)$  if  $\ell \in B$  and  $m(A \times B) = 0$  otherwise.

**Proposition 4.** Consider  $\ell \in \Delta_b(C)$  and  $d, d' \in D_0^*$ . Then it holds that

$$d \geq_C d' \geq_C d^{iid}(\ell) \implies d \geq_B d' \geq_B d^{iid}(\ell).$$

*Proof.* See the Appendix. □

This proposition establishes formally the main trade-off described in the introduction: increasing persistence in consumption risks to an “iid” lottery provides more information about future consumption. We can define correlation aversion as aversion towards increasing correlation to an “iid” temporal lottery.

**Axiom 2.**  $\succ$  exhibits correlation aversion if for every  $d, d' \in D_0^*$  and  $\ell \in \Delta_b(C)$

$$d \geq_C d' \geq_C d^{iid}(\ell) \implies d^{iid}(\ell) \succ_0 d' \succ_0 d.$$

The next result characterizes correlation averse preferences in terms of risk attitudes, under the assumption of UPI, i.e. when there is a trade-off between intertemporal hedging and non-instrumental information.

**Theorem 1.** Consider  $\phi \in \mathcal{C}^2$  that is concave and satisfies UPI. Then every  $\succ$  with KP representation  $(\phi, u, \beta)$  exhibits correlation aversion if and only if  $\phi$  satisfies IRRA.

*Proof.* See the Appendix. □

In words, if  $\phi$  satisfies IRRA, then any KP preference  $(\phi, u, \beta)$  will exhibit correlation aversion; conversely, if  $\phi$  is such that every KP preference  $(\phi, u, \beta)$  exhibits correlation aversion, then  $\phi$  must satisfy IRRA. Moreover, this result implies that indifference to correlation occurs only under risk neutrality, i.e.  $\phi(x) = x$ .<sup>10</sup>

IRRA is one of the most important classes of utility functions (e.g., see [Arrow \(1971\)](#), p. 96). This class notably includes the Epstein-Zin and Hansen-Sargent preferences. Moreover, empirical findings support DARA and IRRA ([Wakker \(2010\)](#), p. 83).

**A bound on preferences for information.** A central implication of [Theorem 1](#) is that IRRA constrains preferences for information. To illustrate, assume that  $\phi$  exhibits HARA. Suppose that for  $x > 1$

$$\phi(x) = \frac{1 - \gamma}{\gamma} \left( \frac{x}{1 - \gamma} + b \right)^\gamma,$$

with  $\gamma < 1$  and  $b \in \left[ \frac{1}{\gamma - 1}, \infty \right)$ . Then, given  $x, y > 1$ , it is immediately possible to check that the local measure of strength of preferences for non-instrumental information

$$-\frac{\phi''(x)}{\phi'(x)} + \beta \frac{\phi''(\beta x + y)}{\phi'(\beta x + y)},$$

is decreasing in the parameter  $b$ . In words, an increase in the parameter  $b$  corresponds to a decrease in the preference for information. In this case, we have

$$R'_\phi(x) = \frac{(\gamma - 1)^2 b}{(b(1 - \gamma) + x)^2},$$

so that IRRA implies that  $b \geq 0$ , thus excluding the case  $b \in \left[ \frac{1}{\gamma - 1}, 0 \right)$ , where the decision maker values information more. Therefore, IRRA limits preferences for information.

More generally, note that IRRA means that  $R_\phi$  is non-decreasing. Therefore, when  $R_\phi$  is differentiable we have:

$$R'_\phi(x) \geq 0 \implies A'_\phi(x) \geq -\frac{A_\phi(x)}{x},$$

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<sup>10</sup>Note that the “boundary” case of indifference to correlation occurs when the relative risk aversion function  $R_\phi$  is constant and equal to zero. Indeed, as a consequence of the proof of [Theorem 1](#), whenever  $R_\phi(x) > 0$  for some  $x$ , one can construct an “iid” lottery that is strictly better than a lottery that differs from it by an IECIT.



for every  $x \neq 0$ . Under DARA it holds  $A'_\phi \leq 0$ , such that we obtain

$$A'_\phi(x) \in \left[ -\frac{A_\phi(x)}{x}, 0 \right].$$

This means that IRRA limits the reduction of absolute risk aversion for a given increase in utility. Therefore, when  $\beta$  is close to unity, IRRA effectively imposes an upper bound on  $-\frac{\phi''(x)}{\phi'(x)} + \beta \frac{\phi''(\beta x + y)}{\phi'(\beta x + y)}$  since

$$-\frac{\phi''(x)}{\phi'(x)} + \beta \frac{\phi''(\beta x + y)}{\phi'(\beta x + y)} \approx -A'_\phi(x)y \leq -\frac{A(x)y}{x}.$$

### 3.1 Implications for asset pricing and long-run risk

The long-run risk model of [Bansal and Yaron \(2004\)](#) is a cornerstone in the consumption-based asset pricing literature for its ability to account for a wide range of asset pricing puzzles. This model relies on [Epstein and Zin's](#) preferences and a consumption process (case I) that satisfies for  $t = 0, \dots$

$$\begin{aligned} \log\left(\frac{c_{t+1}}{c_t}\right) &= m + x_{t+1} + \sigma\epsilon_{c,t+1}, \\ x_{t+1} &= ax_t + \varphi\sigma\epsilon_{x,t+1}, \\ \epsilon_{c,t+1}, \epsilon_{x,t+1} &\sim \text{iid } N(0, 1), \end{aligned} \tag{7}$$

where  $d_t := \log\left(\frac{c_{t+1}}{c_t}\right)$  denotes consumption growth.

Such a model faces the trade-off discussed previously.<sup>11</sup> An investor with recursive preferences values both early resolution of uncertainty and intertemporal hedging, with intertemporal hedging being more valuable due to Epstein-Zin preferences satisfying IRRA. Proposition S.2 in the Supplemental Appendix formalizes this point, showing how aversion to long-run risk emerges from correlation aversion.

It follows that the persistent component of consumption inflates the equity premium because of correlation aversion *in spite of* preferences for non-instrumental in-

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<sup>11</sup>While the present paper has focused on consumption levels, when  $u$  is isoelastic (as in most applications), the same considerations on correlation aversion apply to consumption growth. For example, when  $u(x) = \log(x)$  we have the identity

$$(1 - \beta) \sum_{t=0}^{\infty} \beta^t \log(c_t) = \log(c_0) + \sum_{t=1}^{\infty} \beta^t \log\left(\frac{c_t}{c_{t-1}}\right).$$

$\sigma$	$\varphi$	$a$	$\beta$	$1 - \alpha$	$\rho$	$x_0$	$\pi$
0.0078	0.044	0	0.998	7.5	0	0	0
0.0078	0.044	0.9790	0.998	7.5	0	0	30%
0.0078	0.044	0.9790	0.998	10	0	0	40%

Table 1: Parameters of the LRR model (see [Epstein et al. \(2014\)](#))

formation.<sup>12</sup> However, Epstein-Zin preferences cannot distinguish between risk aversion, correlation aversion, and preferences for non-instrumental information. Indeed, all these properties are reduced to the inequality  $\alpha < \rho$ . More in general, the same holds for preferences which satisfy strong correlation aversion, as I show in Proposition S.3. This fact creates empirical puzzles, which I discuss next.

**The persistence premium.** [Epstein et al. \(2014\)](#) suggest that the long-run risk model entails implausibly high levels of preferences for early resolution of uncertainty. They introduce the concept of a “timing premium” to reflect preferences for early resolution of uncertainty. Under the standard parameters of the model from the literature, they note that the resulting timing premium seems excessively high compared to introspective assessments.

In light of my analysis of correlation aversion, I ask a different question: “What fraction of your wealth would you give up to remove all persistence in consumption?” Formally, define *the persistence premium* by

$$\pi = 1 - \frac{V_0(d^{corr})}{V_0(d^{iid})},$$

where  $d^{iid}$  and  $d^{corr}$  are given by (7) with  $a = 0$  (no persistence) and  $a = 0.9790$ , respectively.

Table 1 summarizes the parameters of the model.<sup>13</sup> Under the level of risk aversion of  $1 - \alpha = 7.5$ , I obtain the persistence premium:  $\pi \approx 30.28\%$ , while we have  $\pi \approx 40\%$  when  $1 - \alpha = 10$ . In other words, an investor with such preferences would be willing to give up either 30% or 40% of his wealth to remove persistence of consumption.

Using the existing experimental evidence from [Andersen et al. \(2018\)](#), my calibra-

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<sup>12</sup>Preferences for early resolution of uncertainty still play a role in other context such as the macroeconomic announcement premium, see for example [Ai and Bansal \(2018\)](#).

<sup>13</sup>This a standard specification for persistence in the literature; see [Bansal and Yaron \(2004\)](#).

tion suggests that we should have at most  $\pi \approx 20\%$  (see Section 6.2.2). This finding implies that either the level of persistence is unrealistic or the degree of risk aversion is too elevated.

**A potential solution.** The previous puzzle and the timing premium puzzle in Epstein et al. (2014) highlight the inability of Epstein-Zin preferences to distinguish risk aversion from correlation aversion and preferences for non-instrumental information. To address this problem, consider the following risk adjustment factor  $\phi_{\gamma,b}$ . For every  $\gamma < 1$  and  $b \geq 0$  define

$$\phi_{\gamma,b}(x) = \frac{1-\gamma}{\gamma} \left( \frac{x}{1-\gamma} + b \right)^\gamma \quad \text{for every } x \in C.$$

Notice that UPI is satisfied, so that By Theorem 1 correlation aversion corresponds to IRRA which is equivalent to  $b \geq 0$ . Moreover, this risk adjustment factor permits a partial separation between risk aversion and preferences for early resolution, meaning that high levels of risk aversion can coexist with both a high or low degree of preferences for early resolution of uncertainty. See Section 6.3 for a formal discussion of these facts. It follows that one can have a low timing premium with a high level of correlation aversion.

The adjustment factor  $\phi_{\gamma,b}$  alone, however, is not enough to disentangle risk aversion from correlation aversion. A well known behavioral feature that introduces correlation aversion is endogeneous discounting. For example, Epstein-Uzawa preferences (see Uzawa 1968 and Epstein 1983) allow the discount factor to depend on the current level of consumption. This reasoning motivates the following recursive representation.

**Definition 8.** Say that  $\succeq$  admits a HARA recursive representation if there exist  $\gamma < 1$  and  $b \geq 0$  such that each  $\succeq_t$  is represented by  $V_t : D_t \rightarrow \mathbb{R}$  such that  $V_T(c) = u(c)$  for every  $c \in C$  and recursively

$$V_t(c, m) = u(c) + \beta(c) \phi_{\gamma,b}^{-1}(\mathbb{E}_m \phi_{\gamma,b}(V_{t+1})) \quad \text{for } t = 0, \dots, T-1,$$

where  $\beta : C \rightarrow (0, 1)$  is continuous and non-increasing,  $u : C \rightarrow \mathbb{R}$  continuous, strictly increasing and satisfies  $u(C) = [0, \infty)$ .

The Epstein-Zin model corresponds to  $b = 0$ ,  $\gamma = \frac{\alpha}{\rho}$ , and to a constant discount function  $\beta : C \rightarrow (0, 1)$ . This extension of Epstein-Zin preferences can partially obtain the desired separation, and therefore could address the puzzle presented in this paper.

### 3.2 Correlation aversion and model misspecification

To further clarify the role of risk attitudes in Theorem 1, I illustrate a tight connection between correlation aversion and fear of model misspecification. Consider the following condition which strengthens IRRA by requiring that the index of relative risk aversion increases sufficiently rapidly.

**Definition 9.** *Say that  $\phi \in \mathcal{C}^4$  satisfies strong correlation aversion (SCA) if it satisfies IRRA and  $R_\phi''(x) \geq 0$  for every  $x \in (0, \infty)$ .*

Thus, SCA requires not only that the index of relative risk aversion  $R_\phi$  is increasing, but also that it increases at a sufficiently fast pace. Observe that both EZ and HS preferences satisfy this condition.

The next key result formalizes the connection between robustness to model misspecification and correlation aversion. Say that a function  $I : X \times X \rightarrow [0, \infty]$  is a generalized (statistical) distance in the sense of Csiszár (1995) if it satisfies  $I(m||\ell) = 0$  if and only if  $m = \ell$  for every  $m, \ell \in X$ .

**Theorem 2.** *Assume that  $\succeq$  admits a KP representation  $(\phi, u, \beta)$  with  $\phi \in \mathcal{C}^4$  that is concave and satisfies UPI. If  $\phi$  satisfies SCA, then  $\succeq$  admits the recursive representation  $(V_t)_{t=0}^2$  given by  $V_2(c) = u(c)$  and*

$$V_t(c, m) = u(c) + \beta \min_{\ell \in \Delta_b(D_{t+1})} \left\{ \mathbb{E}_\ell V_{t+1} + I_{(\phi, u, \beta)}^t(\ell||m) \right\} \quad \text{for } t = 0, 1,$$

where  $I_{(\phi, u, \beta)}^t(\cdot, \cdot) : \Delta_b(D_{t+1}) \times \Delta_b(D_{t+1}) \rightarrow [0, \infty]$  is a generalized distance.

*Proof.* See the Appendix. □

The interpretation is that the decision-maker is concerned about misspecification of the distribution of future consumption. Therefore, alternative distributions are considered based on their distance from  $m$ , as measured by the statistical distance  $I_{\phi, u, \beta}^t$ . The quantity  $I_{\phi, u, \beta}^t(\ell||m)$  measures the “cost” paid when considering the alternative distribution  $\ell$ . When  $\phi(x) = -e^{-\frac{x}{\theta}}$ ,  $I_{\phi, u, \beta}^t$  is given by Relative Entropy (see Strzalecki 2011), that is

$$I_{\phi, u, \beta}^t(\ell||m) = I_\theta^t(\ell||m) = \theta \left( \mathbb{E}_m \left[ \frac{d\ell}{dm} \log \left( \frac{d\ell}{dm} \right) \right] \right),$$

when  $\ell \ll m$  and  $I_{\phi,u,\beta}^t(\ell|m) = \infty$  otherwise. Theorem 2 implies that this interpretation in terms of model misspecification applies to all preferences satisfying strong correlation aversion.

Similar to the variational preferences in [Maccheroni et al. \(2006\)](#), these cost functions can be interpreted as a measure of aversion to model misspecification, or equivalently as an index of correlation aversion. A lower value of each  $I_{\phi,u,\beta}^t$  indicates a higher degree of correlation aversion exhibited by the decision-maker, meaning that considering alternative distributions of future consumption becomes less costly.<sup>14</sup>

To illustrate, consider the common parametrization of Epstein-Zin used in asset pricing with an intertemporal rate of substitution greater than unity  $\frac{1}{1-\rho} > 1$  and  $\alpha < 0$  (see [Bansal and Yaron 2004](#)). As shown in the proof of the theorem, by setting  $q = \frac{\alpha}{\alpha-\rho} > 0$  in this case we have the cost function

$$I_{\phi,u,\beta}^t(\ell|m) = [\mathbb{E}_\ell V_{t+1}] \left[ e^{\frac{1-q}{q} R_q(\ell|m)} - 1 \right] \text{ if } \ell \ll m,$$

and  $I_{\phi,u,\beta}^t(\ell|m) = \infty$  otherwise, where  $R_q(\ell|m) = \frac{1}{q-1} \log \left( \mathbb{E}_m \left[ \left( \frac{d\ell}{dm} \right)^q \right] \right)$  is the Rényi divergence. The Rényi divergence has applications in a variety of fields, including information theory, statistics, and machine learning (see [Sason \(2022\)](#) for a review).

## 4 Discussion of related literature

**Non-EU and Stochastic impatience.** [DeJarnette et al. \(2020\)](#) and [Dillenberger et al. \(2020\)](#) study stochastic impatience, a property that extends impatience to uncertain environments. Like correlation aversion, stochastic impatience is a normatively desirable behavioral postulate. They find that EZ and HS models exhibit stochastic impatience provided that the level of risk aversion is not excessively high relative to the inverse elasticity of intertemporal substitution. The relationship between correlation aversion and stochastic impatience is represented in Figure 4. In particular, correlation aversion can be compatible with stochastic impatience. Similar to my findings, their results also advocate for a more general specification of preferences in order to reduce the level of risk aversion used in applications.

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<sup>14</sup>The general formulation of each  $I_{\phi,u,\beta}^t$  is discussed in the Appendix. The multiplicity of cost functions arises because the setting is not fully stationary when there is a finite horizon. In the Supplemental Appendix, I show that there is a unique cost function when the horizon is infinite.

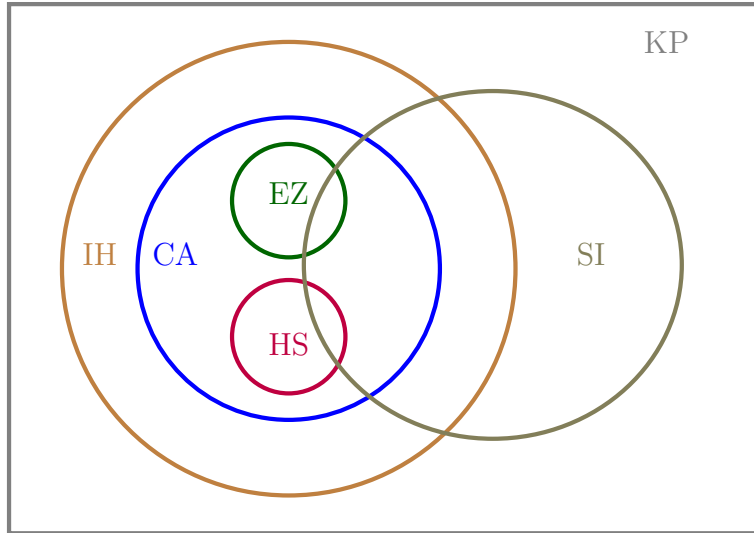


Figure 4: Relationship between correlation averse (CA) preferences and recursive preferences that satisfy intertemporal-hedging (IH), Epstein-Zin (EZ) preferences, and stochastic impatience (SI)

**Climate policy.** [Cai and Lontzek \(2019\)](#) develop a dynamic stochastic general equilibrium model to estimate the effect of economic and climate risks on the social cost of carbon (SCC). They consider productivity shocks that exhibit persistence, leading to consumption growth rates that display long-run risk as in (7). Combined with Epstein-Zin preferences, the inclusion of persistent productivity shocks results in substantially higher social cost of carbon compared to scenarios without productivity shocks (see pp. 2705-2706 in [Cai and Lontzek 2019](#)). My analysis implies that these estimates rely on the fact that Epstein-Zin preferences exhibit correlation aversion rather than preferences for non-instrumental or irrelevant information. This distinction improves the credibility of climate models.

**Utility smoothing and fiscal hedging.** [Karantounias \(2018, 2022\)](#) demonstrates that standard Ramsey tax-smoothing prescriptions for optimal fiscal policy are significantly altered when the decision maker has Epstein-Zin recursive preferences. The planner adopts a fiscal hedging policy to mitigate income shocks: taxing less during unfavorable conditions and more during favorable conditions. A key driver of this result is that with recursive preferences the planner is averse to volatility in future utilities (see [Karantounias \(2018\)](#), p. 2284).

Correlation aversion is a behavioral formulation of this property. Consider again gambles  $A$  and  $B$  described in the introduction. In gamble  $B$ , at  $t = 0$  future utility is constant, while for gamble  $A$  future utility is volatile. Thus, preferring  $B$  to  $A$  indicates aversion to volatility in future utility.

An important implication of the previous results is that such a feature of preferences emerges *in spite* of the fact that recursive preferences value early resolution of uncertainty. Instead, this feature emerges from correlation aversion. As shown by Theorems 1 and 2, aversion to volatility in future utilities—mathematically reflected by concavity of the certainty equivalent—is characterized by bounds on preferences for early resolution of uncertainty. The findings of my paper demonstrate that the same implications for optimal fiscal policy may not hold when using recursive preferences that do not satisfy correlation aversion, as is the case with preferences that exhibit DRRA.

## 5 Concluding remarks

This paper has explored the relationship between non-instrumental information and intertemporal hedging in the context of recursive preferences. I have shown that under reasonable restrictions on risk attitudes, preferences value intertemporal hedging more than non-instrumental information. Correlation aversion requires putting a bound on preferences for non-instrumental information.

I have discussed the importance of this novel trade-off in asset pricing. Note that this trade-off may not be driven solely by risk aversion, as other features of preferences may also be at play. However, in standard models, it is only risk aversion that affects correlation aversion. Further research is necessary to develop models of decision making that enable a greater disentangling. This paper has suggested a potential solution by integrating the Kreps-Porteus recursive framework with time non-separable preferences.

## 6 Appendix

### 6.1 Proofs

#### 6.1.1 Proof of Proposition 1

*Outline of the proof.* The key idea behind the proof is to demonstrate, using established results from information economics (e.g., Theorem 4 in [Kihlstrom 1984](#)), that  $\succeq$  has a preference for information if and only if all functions  $U_t : \Delta_s(D_{t+1,s}) \rightarrow \mathbb{R}$  defined by

$$U_t(m) = \phi\left(u(\bar{c}) + \beta\phi^{-1}\left(\mathbb{E}_m\phi(V_{t+1})\right)\right) \quad \text{for every } m \in \Delta_s(D_{t+1,s}), \quad (8)$$

are convex for every  $\bar{c} \in C$  and  $t = 1, \dots, T-2$ . Straightforward calculations show that convexity of each  $U_t$  is equivalent to (6).

**Lemma 1.** *Each  $U_t$  defined in (8) is convex if and only if (6) holds.*

*Proof.* First I claim that each  $U_t$  defined in (8) is convex if and only if the function  $\Phi : \phi(u(C)) \rightarrow \mathbb{R}$  defined by  $x \mapsto \phi(\bar{c} + \beta\phi^{-1}(x))$  is convex. To see this point, observe that for every  $\bar{c} \in C$  we have that

$$\begin{aligned} U_t(\alpha m + (1-\alpha)m') &\leq \alpha U(m) + (1-\alpha)U(m') \iff \\ &\phi\left(\bar{c} + \beta\phi^{-1}\left(\alpha\mathbb{E}_m\phi(V_{t+1}) + (1-\alpha)\mathbb{E}_{m'}\phi(V_{t+1})\right)\right) \leq \\ &\alpha\phi\left(\bar{c} + \beta\phi^{-1}\left(\mathbb{E}_m\phi(V_{t+1})\right)\right) + (1-\alpha)\phi\left(\bar{c} + \beta\phi^{-1}\left(\mathbb{E}_{m'}\phi(V_{t+1})\right)\right). \end{aligned}$$

Since  $u(C)$  is unbounded above and the statement above must hold for every  $m, m' \in \Delta_s(D_{t+1,s})$  it follows that convexity of  $U_t$  is equivalent to

$$\phi\left(\bar{c} + \beta\phi^{-1}(\alpha x + (1-\alpha)y)\right) \leq \alpha\phi\left(\bar{c} + \beta\phi^{-1}(x)\right) + (1-\alpha)\phi\left(\bar{c} + \beta\phi^{-1}(y)\right),$$

for every  $x, y \in \phi(u(C))$  which is equivalent to convexity of  $\Phi$  for every  $\bar{c} \in u(C)$ . Finally, the claim follows by using Lemma 3 in [Strzalecki \(2013\)](#).  $\square$

*Proof of Proposition 1.* Now if  $\phi$  satisfies (6), then  $U_t$  is convex by Lemma 1. Take  $d, d' \in S$  such that  $d = (c, m), d' = (c, m')$  and  $d \geq_B d'$ . By Theorem 4 in [Kihlstrom \(1984\)](#),  $W_t(\text{marg}_{\Delta_s(D_{t+1,s})} m) \geq W_t(\text{marg}_{\Delta_s(D_{t+1,s})} m')$  for every real-valued convex function  $W_t : \Delta_s(D_{t+1,s}) \rightarrow \mathbb{R}$ . By convexity of  $U_t$ , it follows that  $U_t(\text{marg}_{\Delta_s(D_{t+1,s})} m) \geq U_t(\text{marg}_{\Delta_s(D_{t+1,s})} m')$ , and therefore that  $d \succeq_0 d'$ .



Conversely, consider  $d, d' \in S$  given by

$$d = (c_0, \alpha(\bar{c}, m_1) \oplus (1 - \alpha)(\bar{c}, m_2)),$$

and

$$d' = (c_0, \bar{c}, \alpha m_1 \oplus (1 - \alpha)m_2),$$

where  $\alpha \in [0, 1]$  and  $V_2(m_1) = x$ ,  $V_2(m_2) = y$ . We have that  $d \succeq_0 d'$  if and only if

$$\alpha\phi(\bar{c} + \beta\phi^{-1}(x)) + (1 - \alpha)\phi(\bar{c} + \beta\phi^{-1}(y)) \geq \phi(\bar{c} + \beta\phi^{-1}(\alpha x + (1 - \alpha)y)).$$

Since the statement has to hold for arbitrary  $x, y \in u(C)$  (recall that  $u$  is unbounded above) and  $\alpha \in [0, 1]$ , it follows that the mapping  $x \mapsto \phi(\bar{c} + \beta\phi^{-1}(x))$  must be convex. Hence an immediate application of Lemma 1 concludes the proof.  $\square$

### 6.1.2 Proof of Proposition 3

*Proof.* Take  $d, d' \in D_{0,s}^*$ . Without loss of generality we can assume  $m_1, m'_1$  have common support  $\{c_1, \dots, c_n\} \subseteq [0, 1]$ , and  $(m_2(\cdot|c_i))_{i=1}^n, (m'_2(\cdot|c_i))_{i=1}^n$  have common support over  $\{c_1, \dots, c_m\} \subseteq [0, 1]$ , with  $c_1 < \dots < c_n$  and  $c_1 < \dots < c_m$ . Let  $g_{ij} = m_2(c_i|c_j)m_1(c_i)$ ,  $f_{ij} = m'_2(c_i|c_j)m'_1(c_i)$  for  $i = 1, \dots, n$  and  $j = 1, \dots, m$ . Likewise, let  $G(c, c') = \sum \sum_{j:c_j \leq c} \sum_{i:c_i \leq c'} g_{ij}$  and  $F(c, c') = \sum \sum_{j:c_j \leq c} \sum_{i:c_i \leq c'} f_{ij}$ . Observe that if  $d$  differs from  $d'$  by an IECIT then  $G$  differs from  $F$  by an elementary correlation-increasing transformation as defined by Epstein and Tanny (1980) (see their Definition 1). By Theorem 1 in Epstein and Tanny (1980) it follows that if  $d \geq_C d'$ , we have  $G \geq F$ , which by using the fact that  $m_1 = m'_1$  one obtains

$$\begin{aligned} G &\geq F \\ \iff m'(c_2 \leq c, c_1 \leq c') &\leq m(c_2 \leq c, c_1 \leq c') \\ \iff m'_2(c_2 \leq c | c_1 \leq c') &\leq m_2(c_2 \leq c | c_1 \leq c'), \end{aligned}$$

for every  $(c, c') \in C \times C$  as desired.  $\square$

### 6.1.3 Proof of Proposition 4

*Proof.* Denote with  $\{c, c', \dots, c_N\}$  the support of  $\ell \in \Delta_s(C)$ . It suffices to show that if  $d' \in D_{0,s}^*$  differs from some  $d^{iid}(\ell) \in D_{0,s}^*$  by an IECIT and  $d \in D_{0,s}^*$  differs from

$d'$  by an IECIT then  $d \geq_B d' \geq_B d^{iid}(\ell)$ . Suppose that  $d'$  differs from  $d^{iid}(\ell)$  by an IECIT. Then for some  $\varepsilon \geq 0$  and  $(c, c')$  it holds that

$$\begin{bmatrix} \ell(c) & \ell(c') & \dots & \ell(c_N) \\ \ell(c) & \ell(c') & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \ell(c) & \ell(c') & \dots & \ell(c_N) \end{bmatrix} = \begin{bmatrix} x & 1-x & 0 & \dots & 0 \\ x & 1-x & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \vdots & \vdots & 0 & 1 \end{bmatrix} \begin{bmatrix} \ell(c) + \frac{\varepsilon}{\ell(c)} & \ell(c') - \frac{\varepsilon}{\ell(c)} & \dots & \ell(c_N) \\ \ell(c) - \frac{\varepsilon}{\ell(c')} & \ell(c') + \frac{\varepsilon}{\ell(c')} & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \ell(c) & \dots & \dots & \ell(c_N) \end{bmatrix},$$

for some  $x \in [0, 1]$ , so that  $d' \geq_B d^{iid}(\ell)$ . Using the same reasoning it is immediately clear that  $d \geq_B d'$ .  $\square$

#### 6.1.4 Proof of Theorem 1

It is enough to prove that for every  $d, d' \in D_{0,s}^*$  and  $\ell \in \Delta_s(C)$ ,

$$d \geq_C d' \geq_C d^{iid}(\ell) \implies d^{iid}(\ell) \succeq_0 d' \succeq_0 d.$$

The statement can be extended to arbitrary elements in  $D_0$  by means of continuity of preferences.<sup>15</sup> I provide first the following preliminary result.

**Lemma 2.** *Consider  $d, d'$  such that  $d'$  differs from some  $d^{iid}(\ell)$  by an IECIT and  $d$  differs from  $d'$  by an IECIT. Then there exists a weakly differentiable function  $U : [0, 1] \rightarrow \mathbb{R}$  such that*

1.  $U(0) = V_0(d)$  and  $U(1) = V(d')$ ;
2.  $\lim_{\varepsilon \rightarrow 0} U'(\varepsilon) \leq 0$  whenever  $d = d^{iid}(\ell)$ ;
3.  $U''(\varepsilon) \geq 0$  for every  $\varepsilon \in (0, 1)$ .

*Proof.* See Section S.5 in the Supplemental Appendix.  $\square$

It is now possible to prove Theorem 1. To this end, given  $\ell \in \Delta_s(C)$ , denote with  $d^{corr}(\ell) = (c, m) \in D_{0,s}^*$  defined by  $m_1 = \ell$  and  $m_2(c|c) = 1$  for every  $c \in \text{supp } \ell$ .

<sup>15</sup>To see this, assume that  $d \geq_C d' \geq_C d^{iid}(\ell)$ . Then there exist sequences  $(d_n)_{n=0}^\infty$ ,  $(d'_n)_{n=0}^\infty$  and  $(d^{iid}(\ell_n))_{n=0}^\infty$ , such that  $\lim_n d_n = d$ ,  $\lim_n d'_n = d'$ ,  $\lim_n d^{iid}(\ell_n) = d^{iid}(\ell)$  and  $d_n \geq_C d' \geq_C d^{iid}(\ell_n)$ . Then  $d^{iid}(\ell) \succeq_0 d' \succeq_0 d$  follows by continuity of preferences in the weak\* topology.

*Proof of Theorem 1.* By Lemma 2, there exists  $U : [0, 1] \rightarrow \mathbb{R}$  such that for some  $q_1, q_2 \in [0, 1]$  with  $q_1 < q_2$  it holds that  $U(0) = V_0(d^{iid}(\ell))$ ,  $U(q_1) = V_0(d')$ ,  $U(q_2) = V_0(d)$ ,  $U(1) = V_0(d^{corr}(\ell))$ ,  $\lim_{\varepsilon \rightarrow 0} U'(\varepsilon) \leq 0$ , and  $U'(\varepsilon) \geq 0$  for every  $\varepsilon \in (0, 1)$  (where derivatives are intended in the weak sense, see Section 8.2 in Brezis 2010).<sup>16</sup> I claim that it also holds that

$$\lim_{\varepsilon \rightarrow 1} U'(\varepsilon) \leq 0.$$

Indeed, we have that for some  $p, q \in (0, 1)$  and  $x, y \in u(C)$  such that  $x \geq y$

$$\begin{aligned} \lim_{\varepsilon \rightarrow 1} U'(\varepsilon) &= \lim_{\varepsilon \rightarrow 1} \frac{\partial}{\partial \varepsilon} \left[ p\phi \left( x + \beta\phi^{-1}(\phi(x)(p + q\varepsilon) + \phi(y)(q - q\varepsilon)) \right) + \right. \\ &\quad \left. q\phi \left( y + \beta\phi^{-1}(\phi(x)(p - p\varepsilon) + \phi(y)(q + p\varepsilon)) \right) \right] \\ &\leq (\phi(x) - \phi(y)) \left( \frac{\phi'((1 + \beta)x)}{\phi'(x)} - \frac{\phi'((1 + \beta)y)}{\phi'(y)} \right) \\ &= (\phi(x) - \phi(y)) \int_y^x \frac{(1 + \beta) \frac{\phi''(z(1 + \beta))}{\phi'(z(1 + \beta))} - \frac{\phi''(z)}{\phi'(z)}}{(\phi'(z))^2} dz \leq 0, \end{aligned}$$

where the last inequality follows by the fact that  $\phi$  satisfies IRRA, upon observing that

$$(1 + \beta) \frac{\phi''(z(1 + \beta))}{\phi'(z(1 + \beta))} - \frac{\phi''(z)}{\phi'(z)} \leq 0 \iff -z(1 + \beta) \frac{\phi''(z(1 + \beta))}{\phi'(z(1 + \beta))} \geq -z \frac{\phi''(z)}{\phi'(z)}.$$

Applying the fundamental theorem of calculus for weak derivatives (see Theorem 8.2 in Brezis 2010), it follows that

$$V(d') - V(d^{iid}) = \int_0^{q_1} U'(\tilde{\varepsilon}) d\tilde{\varepsilon} \leq 0,$$

and

$$V(d') - V(d) = \int_{q_1}^{q_2} U'(\tilde{\varepsilon}) d\tilde{\varepsilon} \leq 0.$$

Hence we obtain  $d^{iid} \succeq_0 d \succeq_0 d'$  for every  $\succeq$  with KP representation  $(\phi, u, \beta)$  as desired.

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<sup>16</sup>By applying Lemma 2, if there is a sequence  $(d_i)_{i=0}^N$  such that each  $d_i$  differs from  $d_{i-1}$  by an IECIT, then one can find  $U : [0, 1] \rightarrow \mathbb{R}$  that is continuous and weakly differentiable by constructing  $(U_i)_{i=1}^N$  using Lemma 2 and setting  $U(x) = U_i(\frac{Nx}{i})$  for  $x \in [\frac{i-1}{N}, \frac{i}{N})$ ,  $i = 1, \dots, N-1$ , and  $U(x) = U_N(x)$  for  $x \in [\frac{N-1}{N}, 1]$ .

Conversely, assume that  $\phi$  does not satisfy IRRA. Then there exists  $z < \bar{z}$  in  $\text{int } u(C)$  such that  $R_\phi$  is non-increasing over the interval  $[z, \bar{z}]$  and  $R_\phi(\bar{z}) < R_\phi(z)$ . Pick  $\beta \in (0, 1]$  such that  $\frac{\bar{z}}{1+\beta} > z$ , and let  $x = \frac{\bar{z}}{1+\beta}$ ,  $y = z$ . Consider  $d^{iid}(\ell)$  where  $\ell(x) = \ell(y) = \frac{1}{2}$ . Let  $d^\varepsilon(\ell) = (c_0, m)$  where  $m_2(x|x) = \ell(x) + \frac{1}{2}\varepsilon$ , and  $m_2(y|y) = \ell(y) + \frac{1}{2}\varepsilon$ . Then  $d^\varepsilon(\ell) \geq_C d^{\varepsilon'}(\ell) \geq_C d^{iid}(\ell)$  for  $\varepsilon \geq \varepsilon'$ .

Now define  $U : [0, 1] \rightarrow \mathbb{R}$  such that  $U(\varepsilon) = V_0(d^\varepsilon(\ell))$ . Applying the same reasoning as in Lemma 2, we obtain  $U''(\varepsilon) \geq 0$  for  $\varepsilon \in (0, 1)$  since  $\phi$  satisfies UPI. Therefore since IRRA is not satisfied we obtain

$$\lim_{\varepsilon \rightarrow 1} U'(\varepsilon) \propto (\phi(x) - \phi(y)) \int_y^x \frac{(1+\beta) \frac{\phi''(z(1+\beta))}{\phi'(z(1+\beta))} - \frac{\phi''(z)}{\phi'(z)}}{(\phi'(z))^2} dz > 0,$$

which implies that for some  $\bar{\varepsilon} < 1$  it most hold that  $U'(\bar{\varepsilon}) > 0$  for every  $\bar{\varepsilon} \in [\bar{\varepsilon}, 1)$ . Hence it follows that

$$V(d^1(\ell)) - V(d^{\bar{\varepsilon}}(\ell)) = \int_{\bar{\varepsilon}}^1 U'(\varepsilon) d\varepsilon > 0,$$

which implies that  $d^1(\ell) \geq_C d^{\bar{\varepsilon}}(\ell) \geq_C d^{iid}(\ell)$  but  $d^1(\ell) \succ_0 d^{\bar{\varepsilon}}(\ell)$ . We can therefore conclude that  $\phi$  must satisfy IRRA as desired.  $\square$

### 6.1.5 Proof of Theorem 2

*Outline of the proof.* Using a general result from Hardy et al. (1952) on certainty equivalents, I show that SCA implies that the certainty equivalent  $\phi^{-1}(\mathbb{E}_m \phi(V_{t+1}))$  is concave in utilities.<sup>17</sup> This result allows us to utilize the Fenchel-Moreau duality theorem, revealing that the certainty equivalent can be represented dually as  $\phi^{-1}(\mathbb{E}_m \phi(V_{t+1})) = \min_\ell \mathbb{E}_\ell V_{t+1} + I_{\phi, u, \beta}^t(\ell || m)$ .

I introduce first some important notation: given a measurable space  $(S, \Sigma)$ ,  $ca(\Sigma)$  is the set of all countably additive elements of the set of charges  $ba(\Sigma)$ , while  $ca_+(\Sigma) = ca(\Sigma) \cap ba_+(\Sigma)$  is its positive cone and  $\Delta(\Sigma)$  is the set of countably additive probability measures. Given  $p \in ba(\Sigma)$ , let  $ba(\Sigma, p) = \{v \in ba(\Sigma) : B \in \Sigma \text{ and } p(B) = 0 \text{ implies } v(B) = 0\}$ . Observe that  $ba(\Sigma, p)$  is isometrically isomorphic (see Dunford and Schwartz (1958), Theorem IV.8.16) to the dual of  $L^\infty(p) := L^\infty(S, \Sigma, \mu)$  and

<sup>17</sup> Cerreia-Vioglio et al. (2011) provide a similar representation under the assumption that  $\phi$  is strictly increasing and concave (see their Theorem 24). However, their result significantly differs from this one because they assume that  $u(C) = (-\infty, \infty)$ . This assumption is typically not satisfied in applications, such as the standard Epstein-Zin case.

$ca(\Sigma, p) = ca(\Sigma) \cap ba(\Sigma, p)$  is (isometrically isomorphic to)  $L^1(p)$  (via the Radon-Nikodym derivative  $\nu \mapsto \frac{d\nu}{dp}$ ).

Turning to the proof of Theorem 1, I first introduce important notions related to quasi-arithmetic certainty equivalent functionals: given  $p \in \Delta(\Sigma)$ , let  $M_{\phi,p} : L^\infty(p) \rightarrow \mathbb{R}$  be defined by

$$\phi^{-1} \left( \int \phi(\xi) dp \right) \text{ for every } \xi \in L^\infty(p).$$

The functional  $M_{\phi,p}$  is well-defined whenever  $\phi$  is continuous and non-decreasing. I provide an important result concerning the concave conjugate  $M_{\phi,p}^*$  of the quasi-arithmetic mean  $M_{\phi,p}$ .

**Lemma 3.** *Assume that  $M_{\phi,p}$  satisfies  $M_{\phi,p}(\xi + k) \geq M_{\phi,p}(\xi) + k$  for every  $\xi \in L^\infty(p)$  and  $k \in \mathbb{R}$ . Then the concave conjugate satisfies  $M_{\phi,p}^*(q) = -\infty$  when  $q \notin \Delta(\Sigma)$ .*

*Proof.* Observe first that by the aforementioned isometry between the dual of  $L^\infty(p)$  and  $ba(\Sigma)$ , the concave conjugate  $M_{\phi,p}^*$  can be seen as a mapping  $ba(\Sigma, p) \rightarrow [-\infty, 0]$  defined by

$$M_{\phi,p}^*(q) = \inf_{\xi \in L^\infty(p)} \int \xi dq - M_{\phi,p}(\xi).$$

Now by assumption,

$$M_{\phi,p}^*(q) = \inf_{\xi \in L^\infty(p)} \int \xi dp - M_{\phi,p}(\xi) \leq \phi^{-1} \left( \inf_{\xi \in L^\infty(p)} \int \xi dp - \int \phi(\xi) dp \right).$$

Therefore, Corollary 2A in Rockafellar (1971) implies that  $M_{\phi,p}^*(q) = -\infty$  whenever  $q \notin ca(\Sigma, p)$ . Further, assume that  $q(S) \neq 1$ . Again by assumption on  $M_{\phi,p}$

$$\int (\xi + b) dq - M_{\phi,p}(\xi + b) \leq \int \xi dq - M_{\phi,p}(\xi) + b(q(S) - 1),$$

for all  $b \in \mathbb{R}$  and so  $M_{\phi,p}^*(q) = -\infty$  as desired.  $\square$

Denote with  $L_+^\infty(p) := \{\xi \in L^\infty(p) : \xi \geq 0\}$  the non-negative orthant of  $L^\infty(p)$ .

**Theorem 3** (See Hardy et al. (1952) Theorem 106, Chudziak et al. (2019) or Gollier (2001)). *Consider  $\phi : \mathbb{R} \rightarrow \mathbb{R}$  strictly increasing, strictly concave, and twice differentiable over  $(0, \infty)$ . Then  $M_{\phi,p}|_{L_+^\infty(p)}$  is concave if and only if  $\frac{1}{A_\phi|_{(0,\infty)}}$  is concave.*

*Proof.* If  $A_\phi$  is convex, it follows that by setting  $L_{s,+}^\infty(p) := \{\xi \in L_{s,+}^\infty(p) : \xi = \sum_{k=1}^n a_k \mathbf{1}_{A_k}, (a_k)_{k=1}^n \in \mathbb{R}_+^n\}$ , one can apply Theorem 1 and Theorem 5 in Chudziak et al. (2019) to show that  $M_{\phi,p}|_{L_{s,+}^\infty(p)}$  is concave. Concavity of  $M_{\phi,p}|_{L_+^\infty(p)}$  follows

by the fact that  $L_{s,+}^\infty(p)$  is dense in  $L_+^\infty(p)$ . Conversely, if  $M_{\phi,p}|L_+^\infty(p)$  is concave then  $M_{\phi,p}|L_{s,+}^\infty(p)$  is also concave, which by Theorem 1 and Theorem 5 in [Chudziak et al. \(2019\)](#) implies that  $A_{\phi|(0,\infty)}$  must be convex.  $\square$

Thanks to Theorem 3, we obtain the following powerful result, which shows that the conjunction of DARA and SCA on  $\phi$  implies the concavity of the quasi-arithmetic mean  $M_{\phi,p}|L_+^\infty(p)$ .

**Corollary 1.** *Assume that  $\phi \in \mathcal{C}^4$  is concave and satisfies UPI over  $(0, \infty)$ . Then  $R_\phi'' \geq 0$  implies that  $M_{\phi,p}|L_+^\infty(p)$  is concave.*

*Proof.* First observe that if  $\phi$  satisfies UPI, then by DARA we have  $A'_\phi \leq 0$ . Further, it is immediately evident that  $\frac{1}{A_\phi}$  is concave whenever

$$A_\phi''(x)A_\phi(x) \geq 2(A'_\phi(x))^2,$$

for every  $x \in (0, \infty)$ . This condition is equivalent to

$$xA_\phi''(x) \geq 2x \frac{(A'_\phi(x))^2}{A_\phi(x)}, \quad (9)$$

for every  $x \in (0, \infty)$ . Since  $R'_\phi \geq 0$ , we obtain that for every  $x \in (0, \infty)$  it holds that

$$A(x) \geq -xA'_\phi(x).$$

From this last condition we obtain that for every  $x \in (0, \infty)$

$$-2A'_\phi(x) \geq 2x \frac{(A'_\phi(x))^2}{A_\phi(x)}. \quad (10)$$

Therefore since

$$R_\phi''(x) = xA_\phi''(x) + 2A'_\phi(x),$$

if  $R_\phi'' \geq 0$  it follows that  $xA_\phi''(x) \geq -2A'_\phi(x)$  which by (10) implies that (9) is satisfied. Hence we conclude that if  $\phi$  satisfies SCA then  $\frac{1}{A_\phi}$  is concave. The result therefore follows by Theorem 3.  $\square$

Now consider  $\succeq$  with KP representation  $(\phi, u, \beta)$ . Without loss of generality, assume  $u(C) = [0, \infty)$ . I now show that letting

$$\hat{\phi}(x) = \begin{cases} \phi(x) & x \geq 0 \\ -\infty & x < 0, \end{cases}$$

then  $M_{\hat{\phi},p}$  is concave if  $\phi$  satisfies SCA.

**Lemma 4.** *If  $\phi : [0, \infty) \rightarrow \mathbb{R}$  satisfies SCA, then  $M_{\hat{\phi}, p}$  is concave.*

*Proof.* By Corollary 1,  $M_{\hat{\phi}, p}|_{L_+^\infty(p)}$  is concave. Now given  $\xi, \xi' \in L^\infty(p)$  and  $\alpha \in [0, 1]$ , if  $M_{\hat{\phi}, p}(\alpha\xi + (1 - \alpha)\xi') = -\infty$  then it must be the case that  $M_{\hat{\phi}, p}(\xi) = -\infty$  or  $M_{\hat{\phi}, p}(\xi') = -\infty$ , so that  $M_{\hat{\phi}, p}(\alpha\xi + (1 - \alpha)\xi') \geq \alpha M_{\hat{\phi}, p}(\xi) + (1 - \alpha)M_{\hat{\phi}, p}(\xi')$ . If  $M_{\hat{\phi}, p}(\alpha\xi + (1 - \alpha)\xi') > -\infty$  and  $M_{\hat{\phi}, p}(\xi) = -\infty$  or  $M_{\hat{\phi}, p}(\xi') = -\infty$ , then  $M_{\hat{\phi}, p}(\alpha\xi + (1 - \alpha)\xi') \geq \alpha M_{\hat{\phi}, p}(\xi) + (1 - \alpha)M_{\hat{\phi}, p}(\xi')$ . Finally, if  $M_{\hat{\phi}, p}(\xi), M_{\hat{\phi}, p}(\xi') > -\infty$  then it must be that  $\xi, \xi' \in L_+^\infty(p)$  so that  $M_{\hat{\phi}, p}(\alpha\xi + (1 - \alpha)\xi') \geq \alpha M_{\hat{\phi}, p}(\xi) + (1 - \alpha)M_{\hat{\phi}, p}(\xi')$  as desired.  $\square$

It is important to observe that both EZ and HS preferences satisfy SCA.

**Corollary 2.** *Assume that  $\phi$  is given by  $\phi(x) = \frac{x^\lambda}{\lambda}$  for  $\lambda < 1$  or  $\phi(x) = -e^{-\frac{x}{\theta}}$  with  $\theta > 0$  for every  $x \in \mathbb{R}_+$ . Then  $M_{\hat{\phi}, p}$  is concave.*

*Proof.* Immediate from Theorem 3.  $\square$

It is now possible to deliver a proof of Theorem 2.

*Proof of Theorem 2.* Given  $(V_t)_{t=0}^T$  from the KP representation, observe that for every  $m_t \in \Delta_b(D_t)$ , where  $\mathcal{D}_t$  is the Borel  $\sigma$ -algebra of  $D_t$ , since each  $V_t : D_t \rightarrow \mathbb{R}$ ,  $t = 0, \dots, T$  is continuous we have  $V_t \in L_+^\infty(D_t, \mathcal{D}_t, m_t) := L_+^\infty(m_t)$ . If  $\phi$  satisfies SCA, then by Lemma  $M_{\hat{\phi}, m_t}$  is concave for each  $t = 0, \dots, T - 1$ . By applying the Fenchel-Moreau Theorem (see Phelps (2009), p. 42) and Lemma 4 it follows that

$$M_{\hat{\phi}, m_t}(\xi) = \inf_{q \in \Delta(\mathcal{D}_t, m_t)} \mathbb{E}_q \xi' - M_{\hat{\phi}, m_t}^*(q) \quad \text{for all } \xi' \in L^\infty(m_t).$$

Now using the isometry between  $ca(\mathcal{D}_t, m_t)$  and  $L^1(m_t)$ , we can write

$$M_{\hat{\phi}, m_t}^*(q) = M_{\phi, m_t}^*(q) = \inf_{\xi \in L_+^\infty(m_t) : \mathbb{E}_{m_t} \frac{dq}{dm_t} \xi = \mathbb{E}_q \xi'} \left\{ \mathbb{E}_q \xi - \phi^{-1}(\mathbb{E}_{m_t} \phi(\xi)) \right\}.$$

By applying Proposition 1 in Frittelli and Bellini (1997) one obtains

$$\begin{aligned} -M_{\hat{\phi}, m_t}^*(q) &= \sup_{\xi \in L_+^\infty(m_t) : \mathbb{E}_{m_t} \left[ \frac{dq}{dm_t} \xi \right] = \mathbb{E}_q \xi'} \left\{ \phi^{-1}(\mathbb{E}_{m_t}[\phi(\xi)]) - \mathbb{E}_{m_t} \left[ \frac{dq}{dm_t} \xi \right] \right\} \\ &= \phi^{-1} \left( \mathbb{E}_{m_t}(\phi \circ \psi) \left( k(q) \frac{dq}{dm_t} \right) \right) - \mathbb{E}_q \xi', \end{aligned}$$

where  $\psi = (\phi')^{-1}$  and  $k(q) \in (0, \infty)$  is the only solution to the equation

$$\mathbb{E}\psi \left( k(q) \frac{dq}{dm_t} \right) dm_t = \mathbb{E}_q \xi'.$$

Hence if for  $t = 0, \dots, T-1$  we set

$$I_{\phi, u, \beta}^t(\ell || m_t) := \begin{cases} \phi^{-1} \left( \mathbb{E}_{m_t}(\phi \circ \psi) \left( k(\ell) \frac{d\ell}{dm_t} \right) \right) - \mathbb{E}_\ell V_{t+1} & \ell \ll m_t, \\ +\infty & \text{otherwise,} \end{cases}$$

then one obtains

$$V_t(c, m_t) = u(c) + \beta \min_{\ell \ll m_t} \left\{ \mathbb{E}_\ell V_{t+1} + I_{\phi, u, \beta}^t(\ell || m_t) \right\}$$

where the infimum is attained because  $\{\ell \in \Delta_b(D_{t+1}) : \ell \ll m_t\}$  is a closed subset of the compact metric space  $\Delta_b(D_{t+1})$  (see [Epstein and Zin \(1989\)](#), p. 962) for  $t = 0, \dots, T-1$ .

Finally, observe that each  $I_{\phi, u, \beta}^t$  is a premetric or generalized distance in the sense of [Csiszár \(1995\)](#). Indeed, it can be shown that  $I^t(\ell || m) = 0$  if and only if  $m = \ell$  by adapting the same arguments as in Remark 8 in [Frittelli and Bellini \(1997\)](#). Further, Proposition 16 in [Correia-Vioglio et al. \(2011\)](#) can be used to show that

$$I_{(\phi_1, u, \beta)}^t(\cdot || m) \leq I_{(\phi_2, u, \beta)}^t(\cdot || m),$$

whenever  $A_{\phi_1} \geq A_{\phi_2}$ .

Further, observe that in the Epstein-Zin case we have (see Section 5.2 in [Frittelli and Bellini 1997](#)) by setting  $q = \frac{\alpha}{\alpha - \rho}$ ,

$$I_{\phi, u, \beta}^t(\ell || m) = \mathbb{E}_\ell V_{t+1} \left\{ \left( \mathbb{E}_m \left[ \left( \frac{d\ell}{dm} \right)^q \right] \right)^{-\frac{1}{q}} - 1 \right\},$$

so that upon noticing that the Rényi divergence is given for any  $q > 0$ ,  $q \neq 1$  (see [Van Erven and Harremos 2014](#)) by

$$R_q(\ell || m) = \frac{1}{q-1} \log \left( \mathbb{E}_m \left[ \left( \frac{d\ell}{dm} \right)^q \right] \right),$$

we obtain that whenever  $\alpha < 0$  and  $\frac{1}{1-\rho} > 1$  it holds that  $q > 0$ ,  $q \neq 1$  so that

$$I_{\phi, u, \beta}^t(\ell || m) = \mathbb{E}_\ell V_{t+1} \left[ e^{\frac{1-q}{q} R_q(\ell || m)} - 1 \right],$$

as desired. □



## 6.2 The persistence premium

### 6.2.1 Long-run risk

We have that (see [Epstein et al. \(2014\)](#), pp. 2684-2685)

$$\log V_0(d^{corr}) = \log c_0 + \frac{\beta}{1-\beta a} x_0 + \frac{\beta}{1-\beta} m + \frac{\alpha}{2} \frac{\beta \sigma^2}{1-\beta} \left( 1 + \frac{\varphi^2 \beta^2}{(1-\beta a)^2} \right),$$

and

$$\log V_0(d^{iid}) = \log c_0 + \beta x_0 + \frac{\beta}{1-\beta} m + \frac{\alpha}{2} \frac{\beta \sigma^2}{1-\beta} (1 + \varphi^2 \beta^2).$$

Therefore, we obtain

$$\pi = 1 - \frac{V(d^{corr})}{V(d^{iid})} = 1 - e^{\frac{\beta}{1-\beta a} x_0 - \beta x_0 + \frac{\alpha}{2} \frac{\beta \sigma^2}{1-\beta} \left( \frac{\varphi^2 \beta^2}{(1-\beta a)^2} - \varphi^2 \beta^2 \right)}.$$

$$\pi = 1 - \exp \left( -6.5 \times 0.998 \times \frac{0.0078^2}{2(1-0.998)} \left( 0.044^2 \times \frac{0.998^2}{(1-0.998 \times 0.979)^2} - 0.044^2 \times 0.998^2 \right) \right) \approx 0.302.$$

$$\pi = 1 - \exp \left( -9 \times 0.998 \times \frac{0.0078^2}{2(1-0.998)} \left( 0.044^2 \times \frac{0.998^2}{(1-0.998 \times 0.979)^2} - 0.044^2 \times 0.998^2 \right) \right) \approx 0.393.$$

Therefore, we have that  $\pi \approx 30\%$  with  $\alpha = 7.50$  and  $\pi \approx 40\%$  with  $\alpha = 10$ .

### 6.2.2 An upper bound on the persistence premium

There is no independent study in the literature that quantifies the persistence premium. To have a sense of a potential calibrated value, I conduct a thought experiment that provides an upper bound for the persistence. The thought experiment is based on the comparison between an “i.i.d.” lottery and a maximally correlated lottery in the sense that there is no other temporal lottery more correlated than it. [Andersen et al. \(2018\)](#) estimate an intertemporal utility function under uncertainty which can be written as

$$V(f) = u^{-1} \phi^{-1} \mathbb{E}_P \left[ \phi \left( \sum_{t=1}^2 \beta^t u(f_t) \right) \right],$$

where  $\beta \approx 0.998$ ,  $\phi(x) = x^{0.68}$  and  $u(x) = x^{0.65}$ .

Given  $x > 0$  and  $n = 2$ , let  $d^{iid}$  be the lottery that pays  $x$  and 0 with probability  $\frac{1}{2}$  each and  $f^{corr}$  the process that pay  $(x, x)$  and  $(0, 0)$  with probability  $\frac{1}{2}$  each.

Therefore,  $d^{corr}$  is maximally correlated in the sense that there is no lottery  $d$  such that  $d \geq_C d^{corr}$  and  $d^{corr} \not\leq_C d$ . In this case the persistence premium is given for every  $x > 0$  by

$$\pi = 1 - \frac{\left(0.5 \left(x^{1-0.35} + \frac{x^{1-0.35}}{1+0.114}\right)^{1-0.32}\right)^{1/(1-0.32)(1-0.35)}}{\left((x^{1-0.35})^{1-0.32} \times 0.5 + \left(\frac{x^{1-0.35}}{1+0.114}\right)^{1-0.32} (1-0.5)\right)^{1/(1-0.32)(1-0.35)}} \approx 1 - 0.8 \approx 0.2.$$

Hence  $\pi \approx 20\%$  provides an upper bound for the persistence premium.

### 6.3 HARA recursive preferences

First observe that for every  $\beta \in (0, 1]$  and  $x, y \geq 0$

$$\begin{aligned} -\frac{\phi''_{\gamma,b}(x)}{\phi'_{\gamma,b}(x)} + \beta \frac{\phi''_{\gamma,b}(\beta x + y)}{\phi'_{\gamma,b}(\beta x + y)} &= \left( \frac{1}{\frac{x}{1-\gamma} + b} - \beta \frac{1}{\frac{\beta x + y}{1-\gamma} + b} \right) \\ &= \left( \frac{1}{\frac{x}{1-\gamma} + b} - \frac{1}{\frac{x + \frac{y}{\beta}}{1-\gamma} + \frac{b}{\beta}} \right) \geq 0, \end{aligned}$$

which implies that UPI is satisfied. Further, we have that for  $x > 0$

$$R_{\phi_{\gamma,b}}(x) = \frac{x}{\frac{x}{1-\gamma} + b} = \frac{1}{\frac{1}{1-\gamma} + \frac{b}{x}}.$$

so that IRRA is satisfied whenever  $b \geq 0$ . Moreover

$$R_{\phi_{\gamma,b}}(x) = \frac{x}{\frac{x}{1-\gamma} + b} = \frac{1}{\frac{1}{1-\gamma} + \frac{b}{x}}.$$

so that SCA is not satisfied unless  $b = 0$ . Now observe that in the EZ case  $b = 0$  so that:

$$-\frac{\phi''_{\gamma,0}(x)}{\phi'_{\gamma,0}(x)} + \beta \frac{\phi''_{\gamma,0}(\beta x + y)}{\phi'_{\gamma,0}(\beta x + y)} = \frac{(1-\gamma)y}{x(\beta x + y)},$$

which implies that if risk aversion goes to infinity, i.e. if  $\gamma \rightarrow -\infty$  then

$$-\frac{\phi''_{\gamma,0}(x)}{\phi'_{\gamma,0}(x)} + \beta \frac{\phi''_{\gamma,0}(\beta x + y)}{\phi'_{\gamma,0}(\beta x + y)} \rightarrow +\infty. \quad (11)$$

Finally, we also have that

$$\lim_{\gamma \rightarrow -\infty} \left( \frac{1}{\frac{x}{1-\gamma} + b} - \frac{1}{\frac{x+y}{1-\gamma} + b} \right) = 0.$$

Hence, high levels of risk aversion are compatible with a small demand for non-instrumental information if we assume “large” values of  $b$ , while by (11) for  $b \approx 0$  one can have high levels of risk aversion compatible with high demand of non-instrumental information.

## 7 Bibliography

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# Supplemental Appendix

This supplemental material contains four parts. Section S.1 explains the difference between my notion of correlation aversion with [Kochov's \(2015\)](#) notion of intertemporal hedging. Section S.2 extends the analysis to an arbitrary finite horizon, while Section S.3 extends the result to the case of an infinite horizon. Section S.4 shows that if KP preferences satisfy strong correlation aversion, then risk aversion cannot be disentangled from correlation aversion. Section S.5 provides proofs of claims made in the main text and in the Appendix.

## S.1 Intertemporal hedging

Consider the temporal lotteries  $d = (c_0, m)$ ,  $d' = (c_0, m') \in D_{0,s}$  where for some  $x, y \in C$  we have  $m_1(x)' = m_1(x) = \frac{1}{2}$ ,  $m_2(x|x) = m_2(y|y) = 1$ , and  $m'_2(y|x) = m'_2(x|y) = 1$ . Figure 1 provides a graphical representation of these two lotteries. The lottery  $d$  is obtained by applying an IECIT with  $\varepsilon = \frac{1}{2}$ . The lotteries  $d$  and  $d'$  have perfect positive and negative correlation, respectively.

We can immediately see that  $d \geq_B d'$  and  $d' \geq_B d$ , meaning that  $d$  and  $d'$  are equally informative. The strict preference for  $d'$  over  $d$ , is referred to as correlation aversion by [Bommier \(2007\)](#) and *intertemporal hedging* by [Kochov \(2015\)](#). I adopt the latter terminology as it reflects the fact that with their being equally informative only hedging considerations affect the evaluations of these two lotteries. The next result demonstrates that intertemporal hedging is equivalent to the concavity of  $\phi$  (i.e., risk aversion).

**Proposition S.1.** *Preferences  $\succeq$  with KP representation  $(\phi, u, \beta)$  satisfies intertemporal hedging if and only if  $\phi$  is concave.*

*Proof.* Observe that intertemporal hedging is equivalent to

$$\frac{1}{2}\phi(x + \beta x) + \frac{1}{2}\phi(y + \beta y) \leq \frac{1}{2}\phi(y + \beta x) + \frac{1}{2}\phi(x + \beta y),$$

for every  $x, y \in u(X)$ . Therefore, the statement follows by a straightforward application Theorem 4(a) in [Epstein and Tanny \(1980\)](#).  $\square$



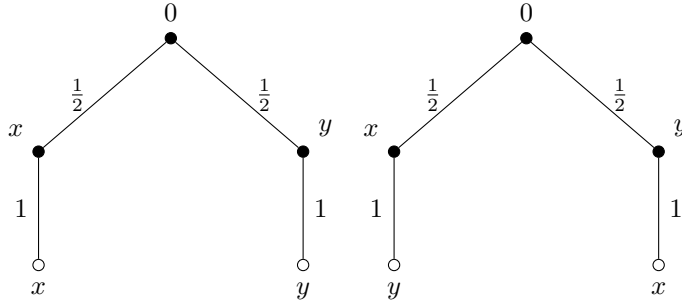


Figure 1: Negative vs positive correlation

## S.2 The case $T < \infty$

The results in Section 3 can be easily extended to an arbitrary horizon  $T < \infty$ . One can define the *present equivalent*  $PE_{\succeq_t}(d)$  of each lottery  $d \in D_t$  as the unique single period consumption level  $c \in C$  such that  $d \sim_t (c, 0, \dots, 0)$ . Now observe that every  $m \in \Delta_b(C \times \Delta_b(D_{t+1}))$  and  $\succeq$  with KP representation  $(\phi, u, \beta)$  induces the probability  $m_{\succeq}$  over  $\Delta_b(C \times \Delta_b(C))$  defined as follows:

$$m_{\succeq}(A \times B) = m(A \times B_{\succeq}) \quad \text{for every closed } A \times B \subseteq C \times \Delta_b(C),$$

where  $B_{\succeq} = \{\ell \in \Delta_b(D_{t+1}) : \ell_{\succeq} \in B\}$  and  $\ell_{\succeq} \in \Delta_b(C)$  is defined by  $\ell_{\succeq}(A) = \ell(\{d \in D_{t+1} : PE_{\succeq_t}(d) \in A\})$ .<sup>1</sup> In words,  $m_{\succeq}$  describes the joint distribution between consumption at time  $t+1$  and the continuation temporal lottery, where each temporal lottery is expressed in terms of one-period consumption. In this way, it is possible to extend the order  $\geq_C$  and the correlation aversion axiom as follows.

**Definition S.1.** Consider  $d = (c, m), d' = (c, m') \in D_0$ . Say that  $d$  is more correlated than  $d'$ , written  $d \geq_C d'$ , if and only if  $(c, m_{\succeq}) \geq_C (c, m'_{\succeq})$ .

Given  $\ell \in \Delta_b(C)$ , the “i.i.d.” lottery is given by  $d^{iid}(\ell) := (c, m)$  where  $m_{\succeq}$  is such that  $m_{\succeq}(A \times B) = \ell(A)$  whenever  $\ell \in B$ , and  $m_{\succeq}(A \times B) = 0$  otherwise.

**Axiom S.1.**  $\succeq$  exhibits correlation aversion if for every  $\ell \in \Delta_b(C)$  and  $d = (c, m), d' = (c, m') \in D_0$  such that  $(c, m_{\succeq}), (c, m'_{\succeq}) \in D_0^*$

$$d \geq_C d' \geq_C d^{iid}(\ell) \implies d^{iid}(\ell) \succeq_0 d' \succeq_0 d.$$

<sup>1</sup>The present equivalent and consequently the lottery  $m_{\succeq}$  are both well defined since preferences are continuous and  $u$  is unbounded above.

We can generalize Theorems 1 and 2 to this setting with an arbitrary finite horizon.

**Theorem S.1.** *Consider  $\phi \in \mathcal{C}^2$  that is concave and satisfies UPI. Then every  $\succeq$  with KP representation  $(\phi, u, \beta)$  exhibits correlation aversion if and only if  $\phi$  satisfies IRRA. Further, if  $\succeq$  that admits a KP representation  $(\phi, u, \beta)$  with  $\phi \in \mathcal{C}^4$  that satisfies SCA, then  $\succeq$  admits the recursive representation  $(V_t)_{t=0}^T$  given by  $V_T(c) = u(c)$  and*

$$V_t(c, m) = u(c) + \beta \min_{\ell \in \Delta_b(D_{t+1})} \left\{ \mathbb{E}_\ell V_{t+1} + I_{(\phi, u, \beta)}^t(\ell || m) \right\} \quad \text{for } t = 0, \dots, T-1$$

where  $I_{(\phi, u, \beta)}^t(\cdot, \cdot) : \Delta_b(D_{t+1}) \times \Delta_b(D_{t+1}) \rightarrow [0, \infty]$  is a generalized distance.

*Proof.* The proof follows the same steps as the proof of Theorems 1 and 2 and is therefore omitted for brevity.  $\square$

The theory presented thus far has focused on studying attitudes towards the correlation between consumption at two separate periods. However, it is also possible to consider more complex patterns of correlation, such as correlation between multiple periods.

To explore this, I introduce a class of ‘‘Markov’’ temporal lotteries, in which the persistence of consumption between periods is determined by a single parameter,  $\varepsilon$ , which ranges from 0 to 1. This parameter is similar to the long-run risk concept introduced by [Bansal and Yaron \(2004\)](#). When  $\varepsilon = 0$ , the lottery outcomes are independent across periods, while for  $\varepsilon = 1$  one has perfect positive correlation. Given  $c_0 \in C$ ,  $\ell \in \Delta_s(C)$  and  $\varepsilon \in [0, 1]$ , define  $d_{\varepsilon, c_0}(\ell)$  recursively as follows:  $d_{T-1, c, \varepsilon}(\ell) = (c, m_{T-1, c, \varepsilon}(\ell))$  where  $m_{T-1, c, \varepsilon} \in \Delta_s(C)$  satisfies

$$m_{T-1, c, \varepsilon}(\ell)(x) = \begin{cases} \ell(c) + (1 - \ell(c))\varepsilon & \text{if } x = c \\ \ell(x) - \ell(x)\varepsilon & \quad x \neq c \end{cases}$$

and recursively for  $2 \leq t \leq T-2$  define  $d_{t-1, c, \varepsilon}(\ell) = (c, m_{t-1, c, \varepsilon}(\ell))$  by

$$m_{t-1, c, \varepsilon}(\ell)(d_{t, x, \varepsilon}(\ell)) = \begin{cases} \ell(c) + (1 - \ell(c))\varepsilon & \text{if } x = c \\ \ell(x) - \ell(x)\varepsilon & \quad x \neq c \end{cases}$$

and finally set  $d_{\varepsilon, c_0}(\ell) = (c_0, m_{1, \varepsilon}(\ell))$  where  $m_{1, \varepsilon}(\ell) := \ell(c)$ . [Figure 2](#) provides a graphical example of a temporal lottery of this type.

The following result demonstrates that, under the assumption of correlation aversion, a higher value of  $\varepsilon$  corresponds to a lower level of utility.

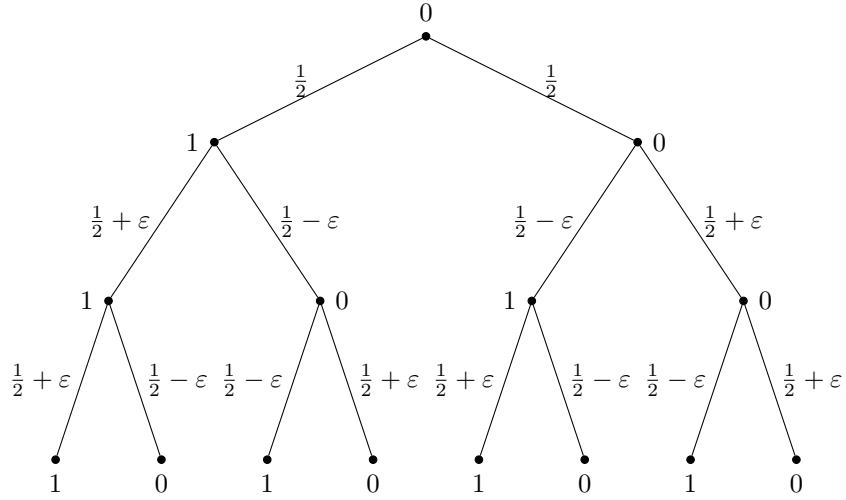


Figure 2: Example of  $d_{\varepsilon,0}(\ell)$  with  $T = 3$  and  $\ell(1) = \ell(0) = \frac{1}{2}$

**Proposition S.2.** Consider  $\phi \in \mathcal{C}^2$  that is concave and satisfies both IRRA and UPI. Consider  $\succeq$  with KP representation  $(\phi, u, \beta)$ . Then for every  $\ell \in \Delta_s(C)$  and  $c_0 \in C, \varepsilon, \varepsilon' \in [0, 1]$

$$\varepsilon \geq \varepsilon' \implies d_{\varepsilon',c_0}(\ell) \succeq_0 d_{\varepsilon,c_0}(\ell).$$

*Proof.* The proof is a straightforward consequence of Theorem 1. Define  $U : [0, 1] \rightarrow \mathbb{R}$  by

$$U(\tilde{\varepsilon}) = \sum_{c \in C} \ell(c) \phi(c + \beta \phi^{-1}(\mathbb{E}_{m_{2,c,\tilde{\varepsilon}}} \phi(V_2))),$$

for every  $\tilde{\varepsilon} \in [0, 1]$  and observe  $d_{\varepsilon',c_0}(\ell) \succeq_0 d_{\varepsilon,c_0}(\ell)$  if and only if  $U(\varepsilon') \geq U(\varepsilon)$ . Notice that by UPI the function  $U$  satisfies  $U''(\tilde{\varepsilon}) \geq 0$  for every  $\tilde{\varepsilon} \in [0, 1]$ . Further observe that by concavity of  $\phi$  we obtain  $\lim_{\varepsilon \rightarrow 0} U'(\varepsilon) \leq 0$  and by IRRA we obtain  $\lim_{\varepsilon \rightarrow 1} U'(\varepsilon) \leq 0$ .<sup>2</sup> Hence for every  $\varepsilon \geq \varepsilon'$  we obtain that

$$\int_{\varepsilon'}^{\varepsilon} U'(\tilde{\varepsilon}) d\tilde{\varepsilon} = U(\varepsilon) - U(\varepsilon') \leq 0,$$

which implies  $d_{\varepsilon',c_0}(\ell) \succeq_0 d_{\varepsilon,c_0}(\ell)$  as desired.  $\square$

<sup>2</sup>The explicit calculations for these results are not presented here, as they mirror those provided in the proof of Theorem 1.

Increasing the value of  $\varepsilon$  involves again a trade-off between non-instrumental information aversion and intertemporal hedging.<sup>3</sup> Under the interpretation that  $\varepsilon \in [0, 1]$  models the persistent component in consumption, the above result establishes a connection between IRRA (and therefore correlation aversion) and aversion to long-run risks.

### S.3 The case $T = \infty$

As the consumption set  $C = [0, \infty) = \mathbb{R}_+$  is identical to that of Epstein and Zin (1989), I follow their approach in introducing the set of temporal lotteries for the case of an infinite horizon, with specific reference to their discussion on pages 940-944. The only deviation in my approach is the use of  $\Delta(X)$  to denote the set of Borel probabilities defined on a metric space  $X$ . The set of temporal lotteries, denoted by  $D(b)$ , is defined in equation 2.3 of their paper and is characterized by the expressions given in equations 2.2-2.11, which define all the relevant objects. I also make use of their characterization of temporal lotteries in  $D(b)$ .

**Theorem S.2** (Theorem 2.2 in Epstein and Zin (1989)). *For every  $b \geq 1$  we have that*

$$D(b) \text{ is homeomorphic to } C \times \hat{\Delta}(D(b)),$$

where

$$\hat{\Delta}(D(b)) := \left\{ m \in \hat{\Delta}(D(b)) : f(m_2) \in \bigcup_{l>0} \Delta(Y(b; l)), \quad m_2 = P_2 m \right\}.$$

Because of this result, each  $d \in D(b)$  can be identified with  $(c, m) \in C \times \hat{\Delta}(D(b))$ . Further, each  $m \in \hat{\Delta}(D(b))$  can be equivalently identified with an element of

$$\hat{\Delta}(C \times \hat{\Delta}(D(b))).$$

Preferences are given by a weak order  $\succeq$  over  $D(b)$ . The utility function  $V : D(b) \rightarrow \mathbb{R}$  is called recursive if it satisfies the following equation for every  $(c, m) \in C \times \hat{\Delta}(D(b))$ ,

$$V(c_0, m) = \left[ c^\rho + \beta \phi^{-1} [(\mathbb{E}_m \phi(V))]^\rho \right]^{1/\rho}, \quad 0 \neq \rho < 1, \quad 0 < \beta < 1, \quad (1)$$

where  $\phi : [0, \infty) \rightarrow \mathbb{R}$ . The next result shows that (1) always has a solution, thus making recursive utility well defined in this context.

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<sup>3</sup>Again, it holds that  $d_{\varepsilon, c_0}(\ell) \geq_B d_{\varepsilon', c_0}(\ell)$  just like in Proposition 4. The proof is omitted for brevity.

**Theorem S.3.** *Suppose that  $\phi$  is concave,  $\rho > 0$  and  $\beta b^\rho < 1$ . Then there exists a continuous  $V : D(b) \rightarrow \mathbb{R}$  that satisfies (1).*

*Proof.* Denote by  $S^+(D(b))$  the set of functions that map from  $D(b)$  into positive real numbers. Let  $h \in S^+(D(b))$  be defined as in p. 963 of Appendix 3 in Epstein and Zin (1989). Further, define  $S_h^+(D(b))$  as follows

$$S_h^+(D(b)) \equiv \left\{ X \in S^+(D(b)) : \|X\|_h \equiv \sup_{d \in D(b)} \frac{X(d)}{h(d)} < \infty \right\}.$$

Define  $T : S_h^+(D(b)) \rightarrow S_h^+(D(b))$  by

$$T(X) = \left[ c^\rho + \beta \phi^{-1} \left[ (\mathbb{E}_m \phi(X))^\rho \right] \right]^{1/\rho} \quad \text{for every } X \in S_h^+(D(b)).$$

Let  $V^*$  be a continuous function such that

$$V^*(c_0, m) = \left[ c^\rho + \beta \left[ \mathbb{E}_m (V^*)^\rho \right] \right]^{1/\rho}, \quad \rho > 0, \quad 0 < \beta < 1,$$

which exists uniquely by Theorem 3.1 in Epstein and Zin (1989) since  $\rho > 0$  and  $\beta b^\rho < 1$ .

Let  $T^0(V^*) = T(V^*)$  and  $T^n(V^*) = T(T^{n-1}(V^*))$ . By Jensen inequality

$$\phi^{-1}(\mathbb{E} \phi(X)) \leq \mathbb{E} X \quad \text{for all } X \in S_h^+(D(b)) \implies T(V^*) \leq V^*.$$

Further, it holds that  $T(V^*) \geq 0$ . By induction, one obtains that the sequence  $(T^n(V^*))_{n=0}^\infty$  is non-increasing and bounded below. Therefore, we can define  $V \in S_h^+(D(b))$  as follows

$$V := \lim_{n \rightarrow \infty} T^n V^*,$$

which is continuous by continuity of  $V^*$  and by the fact that  $T$  maps continuous functions into continuous functions. I now claim that  $V$  solves (1). Since

$$T^n V^*(c_0, m) = \left[ c^\rho + \beta \phi^{-1} \left[ \left( \mathbb{E}_m \phi \left( T^{n-1} V^* \right) \right)^\rho \right] \right]^{1/\rho} \quad \text{for every } m \in D(b),$$

the statement follows by the fact that

$$\begin{aligned} \lim_{n \rightarrow \infty} \phi^{-1} \left[ \left( \mathbb{E}_m \phi \left( T^{n-1} V^*(m) \right) \right)^\rho \right] &= \phi^{-1} \left[ \left( \mathbb{E}_m \phi \left( \lim_{n \rightarrow \infty} T^{n-1} V^*(m) \right) \right)^\rho \right] \\ &= \phi^{-1} \left[ \left( \mathbb{E}_m \phi(V) \right)^\rho \right]. \end{aligned}$$

□

Consider now a weak order  $\succeq$  over  $D(b)$ . Say that  $\succeq$  admits a KP representation  $(\phi, \rho, \beta)$  if there exists  $V : D(b) \rightarrow \mathbb{R}$  that satisfies (1) and such that represents  $\succeq$ . For every  $d \in D(b)$  one can define the present equivalent  $PE_{\succeq}(d)$  as the unique single period consumption level  $c \in C$  such that  $d \sim (c, \mathbf{0})$ , where  $\mathbf{0} \in D(b)$  is the temporal lottery that pays the constant zero level of consumption at every time period. Now observe that every  $m \in \hat{\Delta}(C \times \hat{\Delta}(D(b)))$  and  $\succeq$  with KP representation  $(\phi, \rho, \beta)$  induce the probability  $m_{\succeq}$  over  $\Delta_b(C \times \Delta_b(C))$  defined as follows:

$$m_{\succeq}(A \times B) = m(A \times B_{\succeq}) \quad \text{for every closed } A \times B \subseteq C \times \Delta_b(C),$$

where  $B_{\succeq} = \{\ell \in \hat{\Delta}(D(b)) : \ell_{\succeq} \in B\}$  and  $\ell_{\succeq} \in \Delta_b(C)$  is defined by  $\ell_{\succeq}(A) = \ell(\{d \in D(b) : PE_{\succeq}(d) \in A\})$ .<sup>4</sup> In words,  $m_{\succeq}$  describes the joint distribution between consumption at time  $t+1$  and the continuation temporal lottery, where each temporal lottery is expressed in terms of one-period consumption. In this way, it is possible to extend the order  $\geq_C$  and the correlation aversion axiom as follows.

**Definition S.2.** Consider  $d = (c, m), d' = (c, m') \in D(b)$ . Say that  $d$  is more correlated than  $d'$ , written  $d \geq_C d'$ , if  $(c, m_{\succeq}) \geq_C (c, m'_{\succeq})$ .

Correlation aversion can then be defined as in the main text.

**Definition S.3.**  $\succeq$  exhibits correlation aversion if and only for every  $d, d' \in D(b)$

$$d \geq_C d' \geq_C d^{iid}(\ell) \implies d^{iid}(\ell) \succeq d' \succeq d.$$

The main results of the paper carry out in the same way. Notice, however, that here there is a unique cost function  $I_{\phi, u, \beta}$ .

**Theorem S.4.** Consider  $\phi \in \mathcal{C}^2$  that is concave and satisfies UPI. Then every  $\succeq$  with KP representation  $(\phi, \rho, \beta)$  exhibit correlation aversion if and only if  $\phi$  satisfies IRRA. Further, if  $\succeq$  that admits a KP representation  $(\phi, \rho, \beta)$  with  $\phi \in \mathcal{C}^4$  that additionally satisfies SCA then  $\succeq$  admits the recursive representation

$$V(c, m) = \left[ c^{\rho} + \beta \left( \min_{\ell \in \hat{\Delta}(D(b))} \left\{ \mathbb{E}_{\ell} V + I_{(\phi, u, \beta)}(\ell || m) \right\} \right)^{\rho} \right]^{\frac{1}{\rho}},$$

where  $I_{(\phi, u, \beta)}(\cdot, \cdot) : \hat{\Delta}(D(b)) \times \hat{\Delta}(D(b)) \rightarrow [0, \infty]$  is a generalized distance.

*Proof.* The proof follows the same steps as the proof of Theorems 1 and 2. □

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<sup>4</sup>The lottery  $m_{\succeq}$  is well defined since preferences are continuous,  $u(x) = x^{\rho}$  and each  $m \in \hat{\Delta}(D(b))$  has compact support.

## S.4 Substitution, risk aversion, and correlation aversion

A key motivation for the study of recursive preferences is that the two distinct aspects of preference—intertemporal substitutability and risk aversion—are not intertwined (see [Epstein and Zin \(1989\)](#) pp. 949-950 and [Chew and Epstein \(1991\)](#), Theorem 3.2). However, EZ preferences cannot differentiate between risk aversion and correlation aversion. Indeed, risk aversion corresponds to  $\alpha < \rho$ , which is equivalent to UPI. Moreover, we have that

$$-\frac{\phi''(x)}{\phi'(x)} + \beta \frac{\phi''(\beta x + y)}{\phi'(\beta x + y)} = \frac{1 - \frac{\alpha}{\rho}}{x} - \beta \frac{1 - \frac{\alpha}{\rho}}{\beta x + y} = \frac{(1 - \frac{\alpha}{\rho})y}{x(\beta x + y)}.$$

Therefore, a higher level of risk aversion implies a greater value of the local measure of preferences for non-instrumental information.

I further show that KP preferences that satisfy SCA cannot differentiate between risk aversion and correlation aversion. To this end, consider preferences  $\succeq^i$ ,  $i = 1, 2$  that admit KP representations  $(\phi_1, u_1, \beta_1)$  and  $(\phi_2, u_2, \beta_2)$ , respectively, with both  $\phi_1$  and  $\phi_2$  satisfying SCA.

**Definition S.4.** *Say that  $\succeq^1$  is more correlation averse than  $\succeq^2$  if*

$$I_{(\phi_1, u_1, \beta_1)}^t(\cdot || m) \leq I_{(\phi_2, u_2, \beta_2)}^t(\cdot || m),$$

for every  $t = 0, \dots, T - 1$ .

Comparative risk aversion can be defined in a similar fashion as in [Chew and Epstein \(1991\)](#).

**Definition S.5.** *Say that  $\succeq^1$  is more risk averse than  $\succeq^2$  if for every  $(c_0, m) \in D_0$*

$$(c_0, m) \succeq_0^2 (c_0, c, \dots, c) \implies (c_0, m) \succeq_0^1 (c_0, c, \dots, c),$$

and

$$(c_0, m) \succ_0^2 (c_0, c, \dots, c) \implies (c_0, m) \succ_0^1 (c_0, c, \dots, c).$$

The next result shows the domain  $D_0$  is not enough to distinguish risk aversion from correlation aversion for recursive preferences satisfying SCA.

**Proposition S.3.** *If  $\succeq^1$  is more risk averse than  $\succeq^2$  then  $\succeq^1$  is more correlation averse than  $\succeq^2$ .*

*Proof.* By the same reasoning as in Theorem 3.2 in [Chew and Epstein \(1991\)](#),  $\succeq^1$  is more risk averse than  $\succeq^2$  if and only if they admit KP representations  $(\phi_1, u_1, \beta_1)$  and  $(\phi_2, u_2, \beta_2)$  such that  $u_1 = u_2$ ,  $\beta_1 = \beta_2$  and  $A_{\phi_1} \geq A_{\phi_2}$ . As established in the proof of Theorem 2, if  $A_{\phi_1} \geq A_{\phi_2}$ , then it follows that

$$I_{(\phi_1, u, \beta)}^t(\cdot || m) \leq I_{(\phi_2, u, \beta)}^t(\cdot || m),$$

for every  $t = 0, \dots, T - 1$ , as desired. □

Therefore, the previous result applies to both EZ and HS preferences since they satisfy SCA.

## S.5 Additional proofs

### S.5.1 Proof of Lemma 2

Write the support of  $m_1$  as  $\{c_1, \dots, c_N\}$  and  $p_i = m_1(c_i)$  for every  $i = 1, \dots, N$ . Let  $x_i = u(c_i)$  for  $i = 1, \dots, N$  and

$$U(\varepsilon) = \sum_{i=1}^N p_i \phi \left( x_i + \beta \phi^{-1} \left( \sum_{j=1}^N p_{ji}^\varepsilon \phi(x_j) \right) \right) \quad \text{for every } \varepsilon \in [0, 1],$$

where for some  $\underline{i}, \underline{j}$  it holds that  $p_{\underline{j}\underline{i}}^\varepsilon = p_{\underline{j}\underline{i}} - p_{\underline{i}\underline{j}}\varepsilon$ ,  $p_{\underline{i}\underline{i}}^\varepsilon = p_{\underline{i}\underline{i}} + p_{\underline{i}\underline{j}}\varepsilon$ ,  $p_{\underline{i}\underline{j}}^\varepsilon = p_{\underline{i}\underline{j}} - p_{\underline{j}\underline{i}}\varepsilon$ ,  $p_{\underline{j}\underline{j}}^\varepsilon = p_{\underline{j}\underline{j}} + p_{\underline{j}\underline{i}}\varepsilon$ , and otherwise  $p_{ji} = m_2(c_j | c_i)$  for every other  $j, i$ . Clearly  $U$  defined in such a way satisfies point (1) in the statement. To prove point (2), observe that in



this case we have that for some  $p, q \in (0, 1)$ ,  $k \in \phi(u(C))$  and  $x, y \in u(C)$  with  $x \geq y$

$$\begin{aligned}
\lim_{\varepsilon \rightarrow 0} U'(\varepsilon) &= \lim_{\varepsilon \rightarrow 0} \frac{\partial}{\partial \varepsilon} \left[ p\phi \left( x + \beta\phi^{-1}(\phi(x)(p + q\varepsilon) + \phi(y)(q - p\varepsilon) + k) \right) + \right. \\
&\quad \left. q\phi \left( y + \beta\phi^{-1}(\phi(x)(p - q\varepsilon) + \phi(y)(q + p\varepsilon) + k) \right) \right] \\
&\leq (\phi(x) - \phi(y)) \lim_{\varepsilon \rightarrow 0} \left[ \frac{\phi'(x + \beta\phi^{-1}(\phi(x)(p + q\varepsilon) + \phi(y)(q - p\varepsilon) + k))}{\phi'(\phi^{-1}(\phi(x)(p + q\varepsilon) + \phi(y)(q - p\varepsilon) + k))} - \right. \\
&\quad \left. \frac{\phi'(y + \beta\phi^{-1}(\phi(x)(q + p\varepsilon) + \phi(y)(p - p\varepsilon) + k))}{\phi'(\phi^{-1}(\phi(x)(q + p\varepsilon) + \phi(y)(p - p\varepsilon) + k))} \right] \\
&= (\phi(x) - \phi(y)) \left[ \frac{\phi'(x + \beta\phi^{-1}(\phi(x)p + \phi(y)q + k))}{\phi'(\phi^{-1}(\phi(x)p + \phi(y)q + k))} - \right. \\
&\quad \left. \frac{\phi'(y + \beta\phi^{-1}(\phi(x)q + \phi(y)p + k))}{\phi'(\phi^{-1}(\phi(x)q + \phi(y)p + k))} \right] \\
&= \frac{(\phi(x) - \phi(y))}{\phi'(\phi^{-1}(\phi(x)q + \phi(y)p + k))} \int_y^x \phi'(z)\phi''(z + \beta\phi^{-1}(\phi(z) + k)) dz \leq 0,
\end{aligned}$$

where the last inequality follows by the fact that  $\phi$  is strictly increasing and concave and that  $x \geq y$ . Now to prove point (3), observe that the functions

$$g_1(\varepsilon) := p_i \phi \left( x_i + \beta\phi^{-1} \left( \sum_{j=1}^N p_{j\bar{i}}^\varepsilon \phi(x_i) \right) \right),$$

and

$$g_2(\varepsilon) := p_j \phi \left( x_i + \beta\phi^{-1} \left( \sum_{j=1}^N p_{j\bar{j}}^\varepsilon \phi(x_i) \right) \right),$$

are convex by Lemma 1 in the main text. Then we obtain

$$\begin{aligned}
U''(\varepsilon) &= \frac{\partial^2}{\partial \varepsilon^2} \left[ p_i \phi \left( x_i + \beta\phi^{-1} \left( \sum_{j=1}^N p_{j\bar{i}}^\varepsilon \phi(x_i) \right) \right) + p_j \phi \left( x_i + \beta\phi^{-1} \left( \sum_{j=1}^N p_{j\bar{j}}^\varepsilon \phi(x_i) \right) \right) \right] \\
&= g_1''(\varepsilon) + g_2''(\varepsilon) \geq 0,
\end{aligned}$$

for every  $\varepsilon \in (0, 1)$  as desired.

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